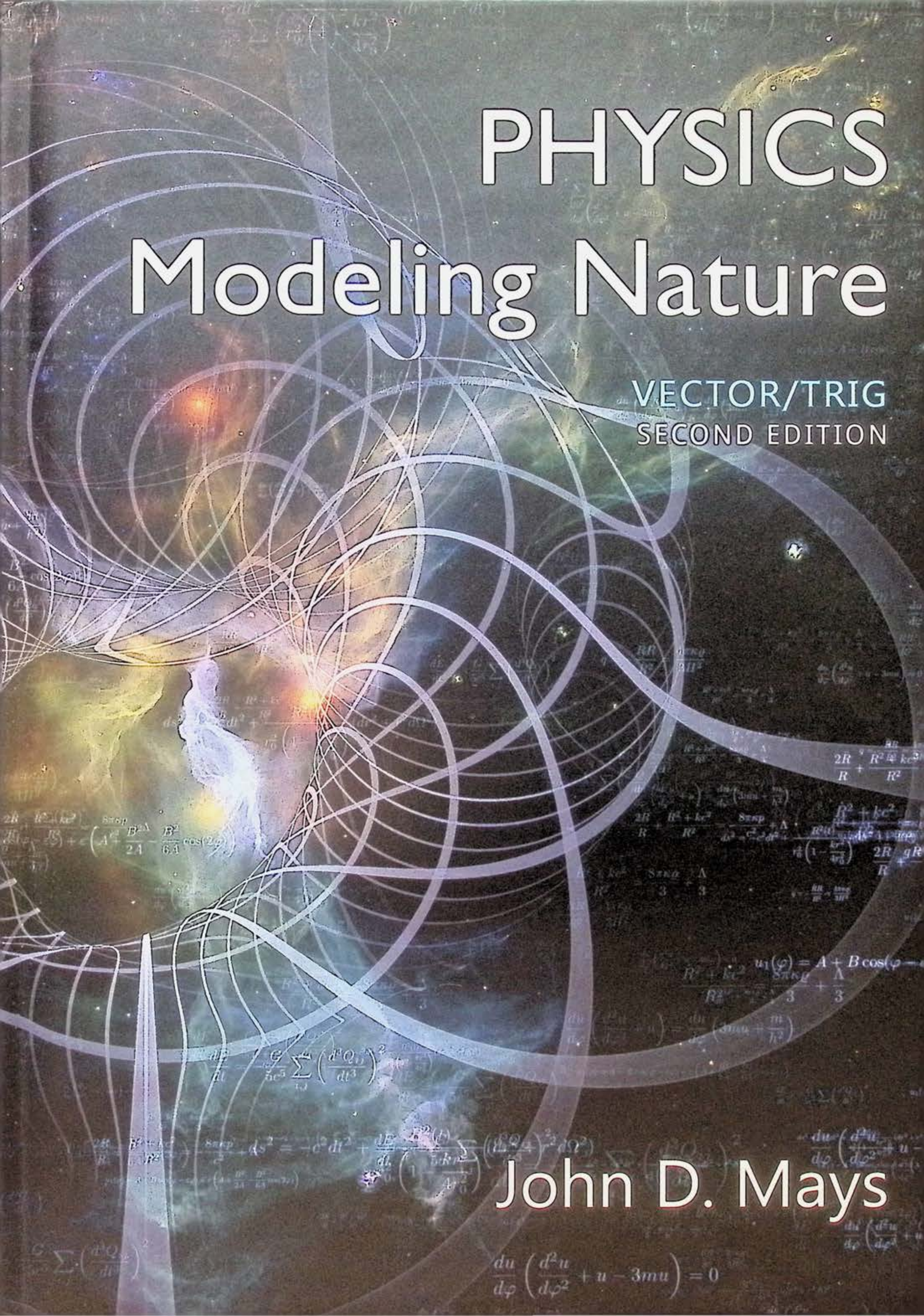
PHYSICS
Modeling Nature

VECTOR/TRIG
SECOND EDITION

John D. Mays



Contents

Physics

Modeling Nature

A Mastery-Oriented Curriculum

Second Edition

The history of science strongly suggests that today's right answer is almost always tomorrow's wrong one; science does not trade in eternal truths, but in temporary approximations.

—Jim Endersby

The scientist is a practical man and his are practical aims. He does not seek the *ultimate* but the *proximate*. The theory that there is an ultimate truth, although very generally held by mankind, does not seem to be useful to science except in the sense of a horizon toward which we may proceed.

—Gilbert N. Lewis

It is wrong to think that the task of physics is to find out how nature *is*. Physics concerns what we can *say* about nature.

—Niels Bohr

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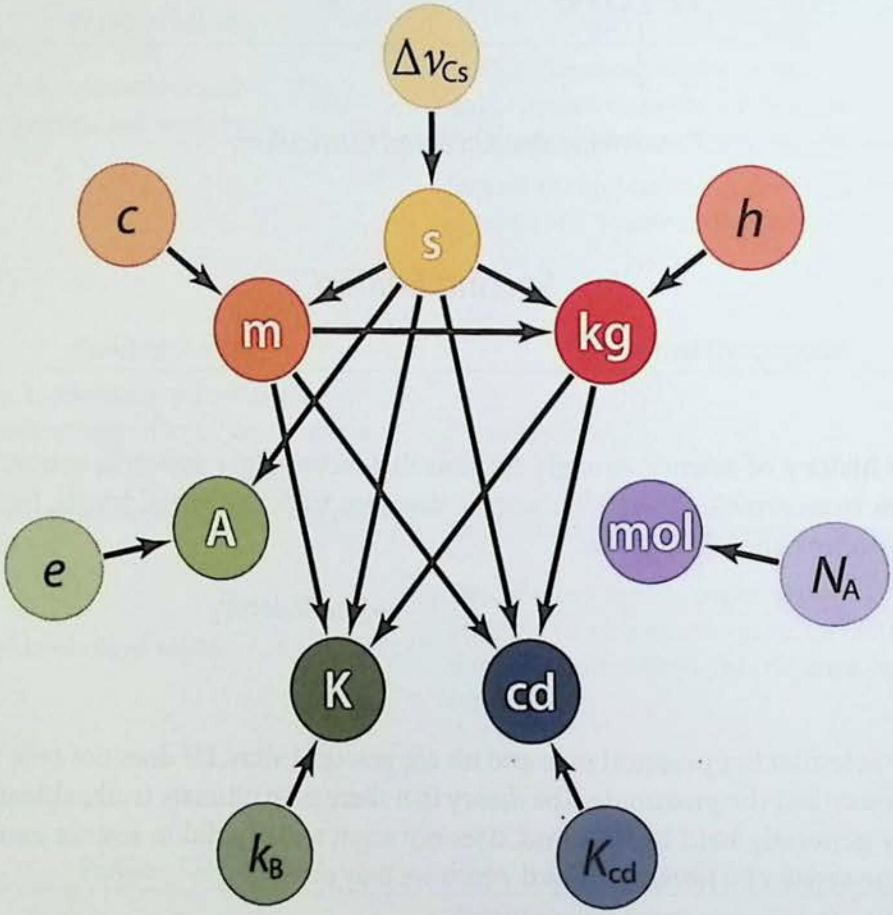
Chapter 1

Mathematical Tools



The SI unit system—or metric system—underwent a serious overhaul in 2019. Prior to the makeover, the kilogram was still defined by a physical object kept in a vault in France. Now, however, the seven base units in the SI system, represented by the circles in the inner ring in the graphic above, are defined in terms of physical constants (the outer ring) and each other. For example, the meter (m) is defined as the distance light travels in 1/299,792,458 seconds. That big number in the denominator is the speed of light in meters per second. So, the definition of the meter involves the speed of light (c) and the definition for the second (s). The arrows in the graphic indicate which units and constants affect others. As you see, the definition of the second affects every other unit definition except one.

Objectives for Chapter 1

After studying this chapter and completing the exercises, you should be able to do each of the following tasks, using supporting terms and principles as necessary.

SECTION 1.1

1. Distinguish between matter and mass.
2. Use standard SI units and prefixes correctly and consistently.
3. Describe the origin of the SI system and state the seven SI base units.
4. Explain what a derived unit is and give examples.
5. Explain what MKS units are and why we use them when solving physics problems.

SECTION 1.2

6. Explain why all measurements contain uncertainty.
7. Distinguish between accuracy and precision.
8. Use significant digits properly when making measurements, interpreting measurements, and performing calculations.
9. Distinguish between random error and systematic error.
10. Describe the meaning of the sample standard deviation.
11. State the equation for the percent difference calculation used in evaluating experimental data.

SECTION 1.3

12. Define *fact*, *theory*, *hypothesis*, and *experiment*.
13. Describe the relationships between facts, theories, hypotheses, and experiments and the roles these play in the process of scientific inquiry.

SECTION 1.4

14. Describe the difference between vector and scalar quantities and give several examples of each.
15. Use the properties of vectors to re-express vector quantities with negative directions or negative angles.
16. Multiply vectors by scalars.
17. Add vector quantities using graphical methods.
18. Calculate rectangular vector components, calculate the magnitude and direction of a vector from its x- and y-components, and use trigonometric methods to calculate vector sums.
19. Compute scalar (dot) and vector (cross) products.[1]
20. Use the right-hand rule to determine the direction of a vector product.

1 This objective may be deferred until needed in Chapters 4 and 5.

1.1 Science and Measurements

1.1.1 No Measurements, No Science

One of the things that distinguishes scientific research from other fields of study is the central role played in science by *measurement.* In every branch of science, researchers study the natural world, and they do it by making measurements. The measurements we make in science are the data we use to quantify the facts we have and to test new hypotheses. These data—our measurements—answer questions such as What is its volume? How fast is it moving? What is its mass? How much time did it take? What is its diameter? What was its frequency and wavelength? When do we expect it to occur again? and many others. Without measurements, modern science would not exist.

Units of measure are crucial in science. Since science is so deeply involved in making measurements, we work with measurements a lot in this course. The value of a measurement is always accompanied by the units of measure—a measurement without its units of measure is a meaningless number. For this reason, your answers to computations in scientific calculations must always show appropriate units.

Before moving on to review units of measure, we review the concepts of matter, volume, and mass.

1.1.2 Matter, Volume, and Mass

The best way to understand mass is to begin with *matter* and its properties. The term *matter* refers to anything composed of atoms or parts of atoms. Your thoughts, your soul, and your favorite song are not matter. You can write down your thoughts in ink, which *is* matter, and your song can be recorded onto a CD, which consists of matter. But ideas and souls are not material and are not made of what we call matter. Another part of this world that is not matter is *electromagnetic radiation*—light, radio waves, x-rays, and all other forms of electromagnetic radiation. Light is a form of energy; it is not matter and it has no mass.

All forms of matter may be described in terms of their physical and chemical properties. Here we focus on just two: all matter takes up space, and all matter has inertia. Describing and comparing these two properties helps make clear what we mean by the term *mass*.

All matter takes up space. Even individual atoms and protons inside of atoms take up space. We *quantify* the amount space occupied by an object by specifying its *volume*. To say that the volume of an object is 1,350 cm^3 (the volume of a typical adult human brain) is to say that to fill a hollow shell this size would require 1,350 neatly stacked cubes, each with a volume of 1 cm^3, illustrated in Figure 1.1.

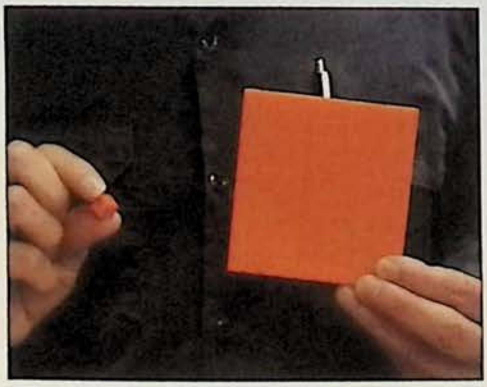


Figure 1.1 Volumes of 1 cm^3 (left) and 100 cm^3 (right).

All matter possesses the property of *inertia*. The effect of this property is that objects resist being accelerated. The more inertia an object has, the more difficult it is to accelerate the object. For example, if the inertia of an object is small, as with a golf ball, the object is easy to accelerate. Golf balls are easy to throw, and if you hit one with a golf club it accelerates at a high rate to a very high speed. But if the amount of inertia an object has is large, as with say, a grand piano, the object is difficult to accelerate. Just try throwing a grand piano or hitting one with a golf club and you will see that it doesn't accelerate at all. This is because the piano has a great deal more inertia than a golf ball.

As with the property of taking up space, we need a way to quantify the inertia of an object. The way we do this is with the variable we call *mass*. The mass of an object is a numerical measurement specifying the amount of inertia the object has. Since inertia is a property of matter, and since all matter is composed of atoms, it should be pretty obvious that the more atoms there are packed into an object, the more mass it has. And since the different types of atoms themselves have different masses, an object composed of more massive atoms has more mass than an object composed of an equal number of less massive atoms.

The basic unit of measure we use to specify an object's mass is the *kilogram*. There are other units such as the gram and the microgram. The kilogram (kg) is one of the base units in the SI unit system, our topic in the next section.

We have established that the mass of an object is a measure of its inertia, which in turn depends on how many atoms it is composed of and how massive those atoms are. The implication of this is that an object's mass does not depend on where it is. A golf ball on the earth has the same mass as a golf ball at the bottom of the ocean, on the moon, or in outer space. Even where there is no gravity, the mass of the golf ball is the same. This is what distinguishes the *mass* of an object from its *weight*.

Weight is caused by the force of gravity acting on an object composed of matter (which we often simply refer to as *a mass*). The weight of an object depends on where it is. An object—or mass—on the moon only weighs about 1/6 of its weight on earth, and in outer space, where there is no gravity, a mass has no weight at all. But the inertia of an object—and thus its mass—does not depend on where it is. This is because an object's mass is based on the matter the object is made of. A Steinway concert grand piano weighs 990 pounds on earth. In outer space, it weighs nothing and will float right in front of you. On the moon it weighs only 165 pounds—about as much as a slim man. But even in deep space, if you try to heave the grand piano—that is, try to accelerate it—the force you feel on your hands will be the same as it would be on the earth or on the moon. This is because the force you feel when you accelerate an object depends on the object's mass.

To summarize in different terms, inertia is a *quality* of all matter; mass is the *quantity* of a specific portion of matter. Inertia is a quality or property all matter possesses. Mass is a quantitative variable, and it specifies a quantity of matter.

1.1.3 The SI Unit System

The *U.S. Customary System* (USCS) is a system of units familiar to everyone raised in the United States. As familiar as this system is with its feet, pounds, and degrees Fahrenheit, this system is not used for scientific work. Thus without further ado, we move on to the SI unit system.

The measurement system universally used for scientific work is the *International System of Units*, known as the *SI system* or the metric system. This system was published in 1960 but originated in France during the French Revolution. The original system included only the meter and the kilogram. Over the years, as measurement treaties were signed and scientific learning advanced, the system grew into the formal SI System that has now been in use since 1960. The SI System is administered by an organization in Sèvres, France (near Paris) known as the International Bureau of Weights and Measures. The SI System has been adopted almost globally. There are only three nations in the world that have not accepted the SI System as their official system of

Unit	Symbol	Quantity
meter	m	length
kilogram	kg	mass
second	s	time
ampere	A	electric current
kelvin	K	temperature
candela	Cd	luminous intensity
mole	mol	amount of substance

Table 1.1. The seven base units in the SI unit system.

Unit	Symbol	Quantity
joule	J	energy
newton	N	force
cubic meter	m^3	volume
watt	W	power
pascal	Pa	pressure

Table 1.2. Some SI System derived units.

measurement: Myanmar, Liberia, and the United States. But even though our road sign markers still give distances in miles, in scientific work the SI System is the one we use.

There are seven *base units* in the SI System, listed in Table 1.1. All other SI units of measure, such as the joule (J) for measuring quantities of energy and the newton (N) for measuring amounts of force, are based on these seven base units. Units based on combinations of the seven base units are called *derived units.* A few common derived units are listed in Table 1.2.

To accommodate measurements of vastly differing size, the SI System uses multipliers on the units of measure to multiply them for large quantities, or to scale them down for smaller quantities. These multipliers are the *metric prefixes.* The complete list of the 20 official SI prefixes is in Table 1.3. You do not need to memorize all of these; some are rarely used. But you do need to memorize some of them. I recommend that all high school science students commit to memory the prefixes listed in Table 1.4.

Multiples	Prefix	deca–	hecto–	kilo–	mega–	giga–	tera–	peta–	exa–	zetta–	yotta–
	Symbol	da	h	k	M	G	T	P	E	Z	Y
	Factor	10	10^2	10^3	10^6	10^9	10^{12}	10^{15}	10^{18}	10^{21}	10^{24}
Fractions	Prefix	deci–	centi–	milli–	micro–	nano–	pico–	femto–	atto–	zetto–	yocto–
	Symbol	d	c	m	μ	n	p	f	a	z	y
	Factor	1/10	$1/10^2$	$1/10^3$	$1/10^6$	$1/10^9$	$1/10^{12}$	$1/10^{15}$	$1/10^{18}$	$1/10^{21}$	$1/10^{24}$

Table 1.3. The SI System prefixes.

I conclude this section with a few brief notes. First, when using the prefixes for quantities of mass, prefixes are never added to the kilogram. Prefixes are only added to the gram, even though the kilogram—not the gram—is the base unit in the SI system. Second, note that when writing the symbols for metric prefixes, the case of the letter matters: *kilo–* always takes a lower-case k, *mega–* always takes an upper-case M, and so on. Third, one of the prefix symbols is not an English letter. The prefix μ for *micro–* is the lower-case Greek letter *mu*, the m in the Greek alphabet. Finally, pay close attention to the difference between multiple prefixes and fraction prefixes. Learning to use the fraction prefixes properly is the most challenging part of mastering the SI System of units, and using them incorrectly in unit conversion factors is a common student error.

Fractions			Multiples		
Prefix	Symbol	Factor	Prefix	Symbol	Factor
centi–	c	$1/10^2$	kilo–	k	10^3
milli–	m	$1/10^3$	mega–	M	10^6
micro–	μ	$1/10^6$	giga–	G	10^9
nano–	n	$1/10^9$	tera–	T	10^{12}
pico–	p	$1/10^{12}$			

Table 1.4. SI System prefixes to commit to memory.

1.1.4 MKS Units

The study of physics is notorious for involving challenging problems. When you are dealing with complex problems involving lots of math, the last thing you want to do is fight your way through a host of unit prefixes and unit conversions to get all the units of measure to work out and agree. For this reason, when solving problems in physics we usually use a subset of the SI system units called *MKS system*. Using the MKS system means using only the SI base units such as the *meter*, the *kilogram*, and the *second* (hence, "MKS") and the units derived directly from the base units.

The wonderful thing about solving problems in MKS units is that any calculation performed with MKS units gives a result in MKS units. This is why the MKS system is so handy and why we use it almost exclusively. There are a few computations that are so simple that conversions to MKS units are not necessary, and I point these out as we go. But for most of the problems you encounter in this text, you should always begin your problem solutions by converting all given quantities into MKS units. Many common conversion factors are listed inside the back cover of the text and in Appendix A. Appendix C lists those you must commit to memory.

1.2 Uncertainty in Measurements

1.2.1 Error and Uncertainty

All measurements contain error because there is no such thing as an exact measurement or a perfect measurement instrument. Any measurement, if made repeatedly with a precise enough instrument, exhibits variation. For this reason, good experimental practice consists of performing measurements repeatedly so that the value under study consists not only of a single measurement but of an entire set of data. Scientists then communicate a measurement by specifying (usually) the mean value and a quantitative description of the *uncertainty* in the measurement. There are several ways to specify measurement uncertainty. The values in Table 1.5 are equivalent ways of expressing the mass of a proton, in units of 10^{-27} kilograms, as it is known today. Note the space inserted after every third decimal place. This is common in values with more than six decimal places and makes them easier to read.

1.672 621 777 ± 0.000 000 074
1.672 621 777(74)
1.672 621 777 ± 0.044 ppm

Table 1.5. Three equivalent ways of expressing the same uncertainty.

In the second of the values in the table (perhaps the most common way of specifying uncertainty), the value in parentheses (74) specifies the uncertainty in the last digit (the 7 in the billionths place). In the third value, the same degree of uncertainty is expressed in "parts per million." To see this, note that one millionth of 1.672 621 777 is 0.000 001 673. Multiplying this by 0.044 gives us 0.044 of these millionths, which is 0.000 000 074. In other words, 0.000 000 074 is 0.044 millionths of 1.672 621 777.

Finally, note that these four expressions of uncertainty are *not* equivalent to saying that the true value of the proton mass is somewhere between $1.672621851 \times 10^{-27}$ kg and $1.672621703 \times 10^{-27}$ kg. They are statistical specifications relating to the amount of variation that occurs when scientists attempt to measure the proton mass. We address this further in Section 1.2.5 below.

1.2.2 Distinguishing Between Accuracy and Precision

The terms *accuracy* and *precision* refer to the practical limitations inherent in making measurements. Science is all about investigating nature, and to do that we must make measurements.

Accuracy relates to error—that is, to the lack of it—which is the difference between a measured value and the true value. The lower the error is in a measurement, the better the accuracy. Error can arise from many different sources including human mistakes, malfunctioning equipment, incorrectly calibrated instruments, vibrations, changes in temperature or humidity, or unknown causes that are influencing a measurement without the knowledge of the experimenter.

Precision refers to the resolution or degree of "fine-ness" in a measurement. The limit to the precision that can be obtained in a measurement is ultimately dependent on the instrument being used to make the measurement. If you want greater precision, you must use a more precise instrument. The degree of precision in every measurement is signified by the measurement value itself because the precision is a built-in part of the measurement. The precision of a measurement is indicated by the number of significant digits (or significant figures) included in the measurement value when the measurement is written down (see below).

Here is an example that illustrates the idea of precision and also helps to distinguish between precision and accuracy. Figure 1.2 is a photograph of a machinist's rule and an architect's scale placed one above the other. Since the marks on the two scales line up consistently, *these two scales are equally accurate*. But the machinist's rule (on top) is more precise. The architect's scale is marked in 1/16-inch increments, but the machinist's rule is marked in 1/64-inch increments. Thus, *the machinist's rule is more precise.*

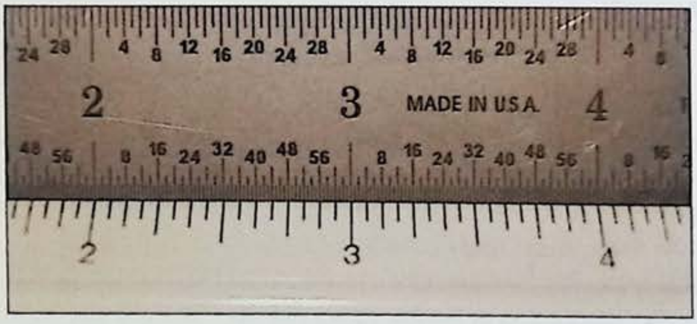


Figure 1.2. The accuracy of these two scales is the same, but the machinist's rule (above) is more precise than the architect's scale (below).

It is important that you are able to distinguish between accuracy and precision. Here is another example to help illustrate the difference. Let's say Theodore and Marius each buy digital thermometers for their homes. The thermometer Theodore buys costs \$10 and measures to the nearest 1°F. Marius pays \$40 and gets one that reads to the nearest 0.1°F. Theodore reads the directions and properly installs the sensor for his new thermometer in the shade. Marius doesn't read the directions and mounts his sensor in the direct sunlight, which causes a significant error in the thermometer reading when the sun is shining on it; thus Marius' measurements are not very accurate. The result is that Theodore has lower-precision, higher-accuracy measurements!

1.2.3 Significant Digits

The precision in any measurement is indicated by the number of *significant digits* it contains. Thus, the number of digits we write in any measurement we deal with in science is very important. The number of digits is meaningful because it shows the precision that was present in the instrument used to make the measurement.

Let's say you are working a computational exercise in a science book. The problem tells you that a person drives a distance of 110 miles at an average speed of 55 miles per hour and wants you to calculate how long the trip takes. The correct answer to this problem *is different* from the correct answer to a similar problem with given values of 110.0 miles and 55.0 miles per hour. And if the given values were 110.0 miles and 55.00 miles per hour, the correct answer would be different yet again. Mathematically, of course, all three answers are going to be the same. If you drive 110 miles at 55 miles per hour, the trip takes two hours. But scientifically, the correct answers to these three problems are different: 2.0 hours, 2.00 hours, and 2.000 hours, respectively. The difference between these cases is in the precision indicated by the given data, which are *measurements*. (Even though this is just a made-up problem in a book and not an actual measurement someone made in an experiment, the given data are still measurements. There is no way to talk about distances or speeds without talking about measurements, even if the measurements are only imaginary or hypothetical.)

So when you perform a calculation with physical quantities (measurements), you can't simply write down all the digits shown by your calculator. The precision inherent in the measurements used in a computation governs the precision in any result you might calculate from those measurements. And since the precision in a measurement is indicated by the number of significant digits, data and calculations must be written with the correct numbers of significant digits. To do this, you need to know how to count significant digits, and you must use the correct number of significant digits in all your calculations and experimental data.

Correctly counting significant digits involves four different cases:

1. Rules for determining how many significant digits there are in a given measurement.
2. Rules for writing down the correct number of significant digits in a measurement you are making and recording.
3. Rules for computations you perform with measurements—multiplication and division.
4. Rules for computations you perform with measurements—addition and subtraction.

We now address each of these cases, in order.

Case 1 We begin with the rule for determining how many significant digits there are in a given measurement value. The rule is as follows:

- *The number of significant digits in a number is found by counting all the digits from left to right, beginning with the first nonzero digit on the left. When no decimal is present, trailing zeros are not considered significant.*

As examples of this rule, consider the following:

15,679 This value has five significant digits.

21.0005 This value has six significant digits.

37,000 This value has only two significant digits because when there is no decimal, trailing zeros are not significant. Notice that the word *significant* here is a reference to the precision of the measurement, which in this case is rounded to the nearest thousand. The zeros in this value are certainly *important*, but they are not *significant* in the context of precision.

0.0105 This value has three significant digits because we start counting digits with the first nonzero digit on the left.

0.001350 This value has four significant digits. Trailing zeros count when there is a decimal.

The significant digit rules enable us to tell the difference between two measurements such as 13.05 m and 13.0500 m. Again, these values are obviously equivalent *mathematically*. But they are different in what they tell us about the process of how the measurements were made—and science deals in measurements. The first measurement has four significant digits. The second measurement is more precise. It has six significant digits, and was made with a more precise instrument.

Now, just in case you are bothered by the zeros at the end of 37,000 that are not significant, here is one more way to think about significant digits that may help. The precision in a measurement depends on the instrument used to make the measurement. If we express the measurement in different units, this cannot change the precision of the value. A measurement of 37,000 grams is equivalent to 37 kilograms. Whether we express this value in grams or kilograms, it still has two significant digits.

Case 2 The second case addresses the rules that apply when you are recording a measurement yourself, rather than reading a measurement someone else has made. When you take measurements yourself, as you do in laboratory experiments, you need to know the rules for which digits are significant in the reading you are taking on the measurement instrument. The rule for taking measurements depends on whether the instrument you are using is a digital instrument or an analog instrument. Here are the rules for these two possibilities:

- *Rule 1 for digital instruments* — *For the digital instruments commonly found in high school or undergraduate science labs, assume all of the digits in the reading are significant except leading zeros.*
- *Rule 2 for analog instruments* — *The significant digits in a measurement include all the digits known with certainty, plus one digit at the end that must be estimated between the finest marks on the scale of your instrument.*

The first of these rules is illustrated in Figure 1.3. The reading on the left has leading zeros, which do not count as significant. Thus, the first reading has three significant digits. The second reading also has three significant digits. The third reading has five significant digits.

Figure 1.3. With digital instruments, all digits are significant except leading zeros. Thus, the numbers of significant digits in these readings are, from left to right, three, three, five, and five.

The fourth reading also has five significant digits because with a digital display, the only zeros that don't count are the leading zeros. Trailing zeros are significant with a digital instrument. However, when you write this measurement down, you must write it in a way that shows those zeros to be significant. The way to do this is by using scientific notation. Thus, the right-hand value in Figure 1.3 must be written as 4.2000×10^4.

Dealing with digital instruments is actually more involved than the simple rule above implies, but the issues involved go beyond what we typically deal with in introductory or intermediate science classes. So, simply take your readings and assume that all the digits in the reading except leading zeros are significant.

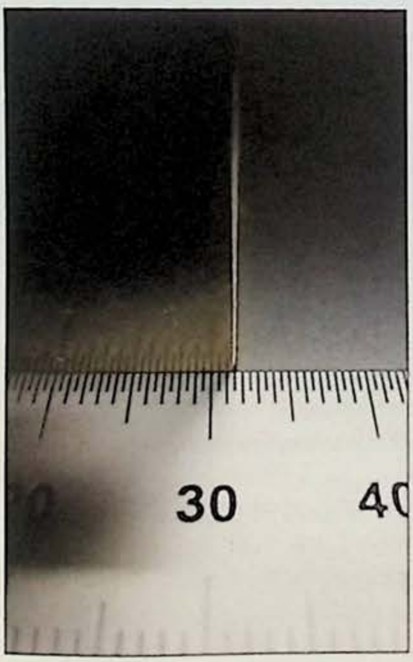


Figure 1.4. Reading the significant digits with a machinist's rule.

Now let's look at some examples illustrating the rule for analog instruments. Figure 1.4 shows a machinist's rule being used to measure the length in millimeters (mm) of a brass block. We know the first two digits of the length with certainty; the block is clearly between 31 mm and 32 mm long. We have to estimate the third significant digit. The scale on the rule is marked in increments of 0.5 mm. Comparing the edge of the block with these marks, I would estimate the next digit to be a 6, giving a measurement of 31.6 mm. Others might estimate the last digit to be 5 or 7; these small differences in the last digit are unavoidable because the last digit is estimated. Whatever you estimate the last digit to be, two digits of this measurement are known with certainty, the third digit is estimated, and the measurement has three significant digits.

The photograph in Figure 1.5 shows a liquid volume measurement in milliliters (mL) being made with a buret. The scale is marked in increments of 0.1 mL. This means we are to estimate to the nearest 0.01 mL. To one person, it may look like the bottom of the meniscus (the black curve) is just below 2.2 mL, so that person would call this measurement 2.21 mL. To someone else, it may seem that the bottom of the meniscus is right on 2.2, in which case that person would call the reading 2.20 mL. Either way, the reading has three significant digits and the last digit is estimated to be either 1 or 0.

Figure 1.5. Reading the significant digits on a buret.

As a third example, Figure 1.6 shows a liquid volume measurement being made in a graduated cylinder. The scale on the graduated cylinder is marked in increments of 1 mL. In the photo, the entire meniscus appears silvery in color with a black curve at the bottom. For the liquid shown in the figure, we know the first two digits of the volume measurement with certainty, because the reading at the bottom of the meniscus is clearly between 82 mL and 83 mL. We have to estimate the third digit, and I would estimate the black line to be at 40% of the distance between 82 and 83, giving a reading of 82.4 mL.

Figure 1.6. Reading the significant digits on a graduated cylinder.

It is important for you to keep the significant digits rules in mind when you are taking measurements and entering data for your lab reports. The data in your lab journal and the values you use in your calculations and report should correctly reflect the use of the significant digits rules as they apply to the actual instruments you use to take your measurements.

Case 3 The third and fourth cases of rules for significant digits apply to the calculations you perform with measurements. In Case 3, we are concerned with multiplication and division. The main idea behind the rule for multiplying and dividing is that the precision you report in your result cannot be higher than the precision that is in the measurements you start with. The precision in a measurement depends on the instrument used to make the measurement, nothing else. Multiplying and dividing things cannot improve that precision, and thus your results can be no more precise than the measurements that went into the calculations. In fact, your result can be no more precise than the *least precise value* used in the calculation. The least precise value is, so to speak, the "weak link" in the chain, and a chain is no stronger than its weakest link.

Here are the two rules for using significant digits in calculations involving multiplication and division:

- *Rule 1* *When multiplying or dividing, count the significant digits in each of the values you will use in a calculation, including any conversion factors you are using. (However,* *note: conversion factors that are exact are not considered.) Determine how many significant digits there are in the least precise of all of these values. The result of your calculation must have this same number of significant digits.*

- Rule 2 *When performing a multi-step calculation, keep at least one extra digit during intermediate calculations, and round to the final number of significant digits you need at the very end. This practice ensures that small round-off errors don't accumulate during a multi-step calculation. This extra digit rule also applies to unit conversions performed as part of the computation.*

Case 4 The fourth case of rules for significant digits also applies to the calculations you perform with measurements. In Case 4, we are concerned with addition and subtraction.

The rule for addition and subtraction is completely different from the rule for multiplication and division. When performing addition, it is not the number of significant digits that governs the precision of the result. Instead, it is the *place value of the last digit that is farthest to the left in the numbers being added* that governs the precision of the result. This rule is quite wordy and is best illustrated by an example. Consider the following addition problem:

$$\begin{array}{r} 13.65 \\ 1.9017 \\ +\ \underline{1{,}387.069} \\ 1{,}402.62 \end{array}$$

Of the three values being added, 13.65 has digits out to the hundredths place, 1.9017 has digits out to the ten thousandths place, and 1,387.069 has digits out to the thousandths place. Looking at the final digits of these three, you can see that the final digit farthest to the left is the 5 in 13.65, which is in the hundredths place. This is the digit that governs the final digit of the result. There can be no digits to the right of the hundredths place in the result. The justification for this rule is that one of our measurements is precise only to the nearest hundredth, even though the other two are precise to the nearest thousandth or ten thousandth. We are going to add these values together, and one of them is precise only to the nearest hundredth. Thus, it makes no sense to have a result that is precise to a place more precise than that, so hundredths are the limit of the precision in the result.

Correctly performing addition problems in science (where nearly everything is a measurement) requires that you determine the place value governing the precision of your result, perform the addition, and then round the result. In the above example, the sum is 1,402.6207. Rounding this value to the hundredths place gives 1,402.62.

1.2.4 Random and Systematic Error

The two main types of error in experimental measurements are *random error* and *systematic error.* Random errors are caused by unknown and unpredictable fluctuations in the experimental setup. Examples of random error would be changes in the apparatus due to temperature fluctuations in the room, vibrations or wind that influence the measurement in a random fashion, or electronic noise that influences the readings in your instruments. When you calculate and discuss the uncertainty in your measurements, you are discussing the random error that caused your measurements to fluctuate randomly around the mean value.

Systematic errors are errors that bias the experimental results in one direction, and are usually caused by equipment defects, miscalibration of measurement instruments, or an experimenter who consistently misreads or misuses the instruments in the same way. Usually, when discussing systematic error, we are talking about problems that could be eliminated by proper use, calibration, and operation of the equipment.

Your percent difference values (Section 1.2.6) can be influenced by factors you did not take into account in your predictions, and the result can be percent difference values that look like they include systematic error. A common example of this is physics experiments involving motion that do not take friction into account. If you make predictions in a mechanical system without taking friction into account, your results will all be biased in the same way relative to your measurements. This is *not* an "experimental error"; it is the result of using approximations in your theoretical modeling of the experiment. However, it is a contributor to your percent difference values and could play a role in the discussion of your results.

1.2.5 Standard Deviation

No doubt, you are already familiar with the statistical parameters *mean* and *median*. In the language of statistics, these parameters are *measures of center*—they indicate where the "middle" of a data set is. The *standard deviation* of a data set is another statistical parameter, one that indicates the amount of "spread" in a data set. There are two different calculations for the standard deviation: the population standard deviation (σ) and the sample standard deviation (s). The population standard deviation applies to data sets that include every member of a population; the sample standard deviation applies to data sets that contain a sample of values from a population, but not the entire population. For example, it is possible to know the age of every single student in a school, and such a data set constitutes a population. But since a data set of scientific measurements almost never contains every possible measurement value that could occur, sets of measurement data are usually samples and the sample standard deviation is the one most commonly used with scientific data.

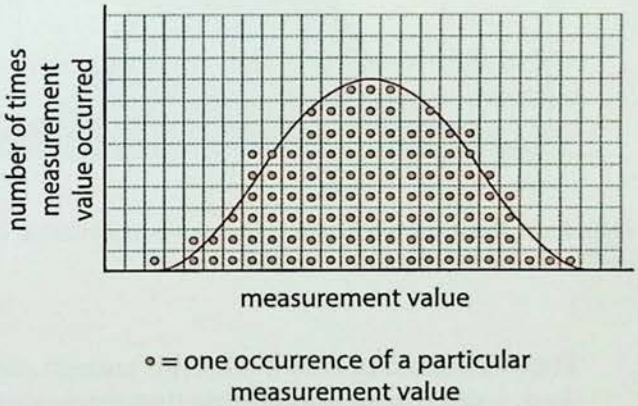


Figure 1.7. A Gaussian distribution forms from the data when a specific measurement is made repeatedly with an instrument precise enough to show the variation.

When taking measurements or performing calculations from repeated trials of an experiment, the values of the measurements—if there are enough of them—often form a *Gaussian distribution* (also called a *normal distribution*). Figure 1.7 illustrates how measurements form a Gaussian distribution. Each circle in the figure represents a measurement. Most of the measurement values are close to the mean value, so the distribution peaks here; some measurements lie far above or below the mean, giving the distribution its "tails."

Figure 1.8 shows two more typical Gaussian distributions, the second distribution more spread out than the first. The standard deviation, s, of a data set is a measure of how spread out the data are. Larger values of s mean wider spread; smaller values of s mean a narrower spread. The more accurate your experimental methods, and

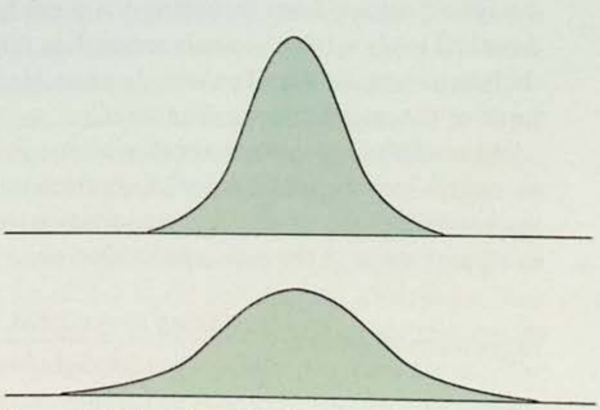


Figure 1.8. Gaussian distributions: narrow, with smaller s (top), and wide, with larger s (bottom).

the more precise your instruments, the narrower the spread in the data should be (all else being equal). Data that are very close together have a very small value of s—exactly what you want. If all the data have exactly the same value, then $s = 0$.

To help you understand the standard deviation a bit more, Figure 1.9 indicates the relationship between the standard deviation of a data set and the shape of the data distribution (assuming a Gaussian distribution). The mean of the distribution is in the center, where the zero is on the horizontal scale. The scale is marked in increments of one standard deviation: $1s$, $2s$, and so on. In a Gaussian data distribution, 68% of the data values lie within one standard deviation of the mean, 95.4% of the data lie within two standard deviations of the mean, and 99.7% of the data lie within three standard deviations of the mean. Note that the units for s are the same as the units of the data. If your data are in mL, s is also in mL. If the mean of the data is 75.5 mL and $s = 3.1$ mL, then 68% of the data lie between 72.4 mL and 78.6 mL. If s is only 1.1 mL, then 68% of the data lie between 74.4 mL and 76.6 mL—a narrower distribution with less *uncertainty* in the measurements.

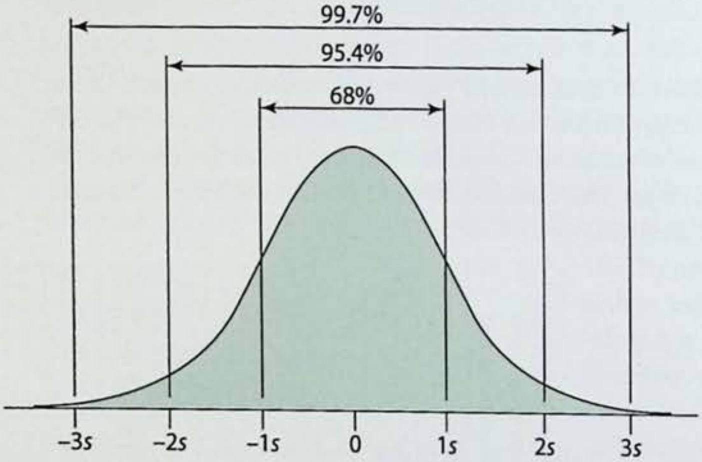


Figure 1.9. Relationship between the standard deviation and the area under the curve for a Gaussian distribution.

The sample standard deviation is often used as a measure of uncertainty in a data set. As I write above, all measurements contain error; that's just a fact of life. All measurements made with enough precision show variation, and together the values form a distribution, indicating that there is uncertainty as to the true value of the parameter being measured. Quoting the value of s for a set of data is a very common way of indicating the uncertainty in the data. All computer spreadsheet applications and graphing calculators can calculate the sample standard deviation. Just enter your data and look up how to run the calculation of the sample standard deviation on your device.

1.2.6 Calculating Percent Difference

The topic in this section applies primarily to the experiments you perform as part of your study of physics. I am including the topic here because it addresses one of the common mathematical tools we use to analyze the data taken in an experiment and compare them to the predictions we make based on our theoretical understanding of how nature works. (We address the topic of theories in the next section.)

One of the conventional calculations in high school and college physics experiments is the so-called "experimental error." Experimental error is typically defined as the difference between the predicted value (which comes from scientific theory) and the experimental value, expressed as a percentage of the predicted value, or

$$\text{experimental error} = \frac{|\text{predicted or accepted value} - \text{experimental value}|}{\text{predicted or accepted value}} \times 100\%$$

Although the term "experimental error" is widely used, it is in my view a poor choice of words. When there is a mismatch between theory and experiment, the experiment may not be the source of the error. Often, it is the theory that is found wanting. This is how science advances.

It is, of course, true that at the introductory and intermediate level students are not generally engaged in research that uncovers weaknesses in scientific theories. At this level, the difference between prediction and experimental result may well be due entirely to "experimental error" arising from experimental limitations or inaccuracies. However, I prefer that students develop scientific habits of mind, and in the real world of scientific research in physics and engineering, the measurements are as accurate as the experimenters know how to make them, and one does not know whether differences between mathematical prediction and experimental result are due to the mathematical model or error in the experiment.

I prefer to use the phrase *percent difference* to describe the value computed by the above equation. When quantitative results are compared to quantitative predictions or accepted values, students should compute the percent difference as

$$\text{percent difference} = \frac{|\text{predicted or accepted value} - \text{experimental value}|}{\text{predicted or accepted value}} \times 100\%$$

One more note. In the study of statistics, there is a calculation called the "percentage difference," in which the difference between two values is divided by their average. To avoid potential future confusion, you should note the distinction between the calculation we are using here and the one arising in statistics.

1.3 Modeling Nature

1.3.1 Science as Mental Model Building

Students usually find the study of physics to be fascinating and rewarding—even though it is also challenging. This is because physics is all about modeling the fundamental interactions of the matter and energy the world is composed of. The fact that humans can accurately model nature with mathematics is a wonderful consequence of God's design: God made the world in such a way that it lends itself to mathematical characterization, and God made human beings with the mental ability to imagine mathematical structures. The world's mathematical properties and our mathematical abilities fit together extremely well, and the result is that over the past 400 years, the accomplishments in the field of physics have been nothing short of stupendous.

Before we dive into modeling nature with mathematics, we need to pause and consider again how the modeling process works in science, and what kind of knowledge science provides for us. It is helpful to think of science as the process of building "mental models" of the natural world. These mental models are called *theories*. The information we use to build our mental models—scientific facts—comes from experiments, observations, and inferences from these.

The theoretical models developed by scientists are the basis for our entire understanding of how the natural world functions. Successful theories are those that account for the facts we know and lead to new hypotheses (predictions) that can be put to the test. It is helpful to think about the relationship between scientific facts, theories, hypotheses, and experiments as illustrated in Figure 1.10. This diagram illustrates what I call the *Cycle of Scientific Enterprise*. You may have studied this diagram before in a previous class. It is important for every student to develop a correct understanding of the kind of knowledge scientific study provides for us. The *goal* of science is to uncover the truth about how nature works, but scientific theories are always works in progress. Even our best theories are provisional and subject to change. For this reason,

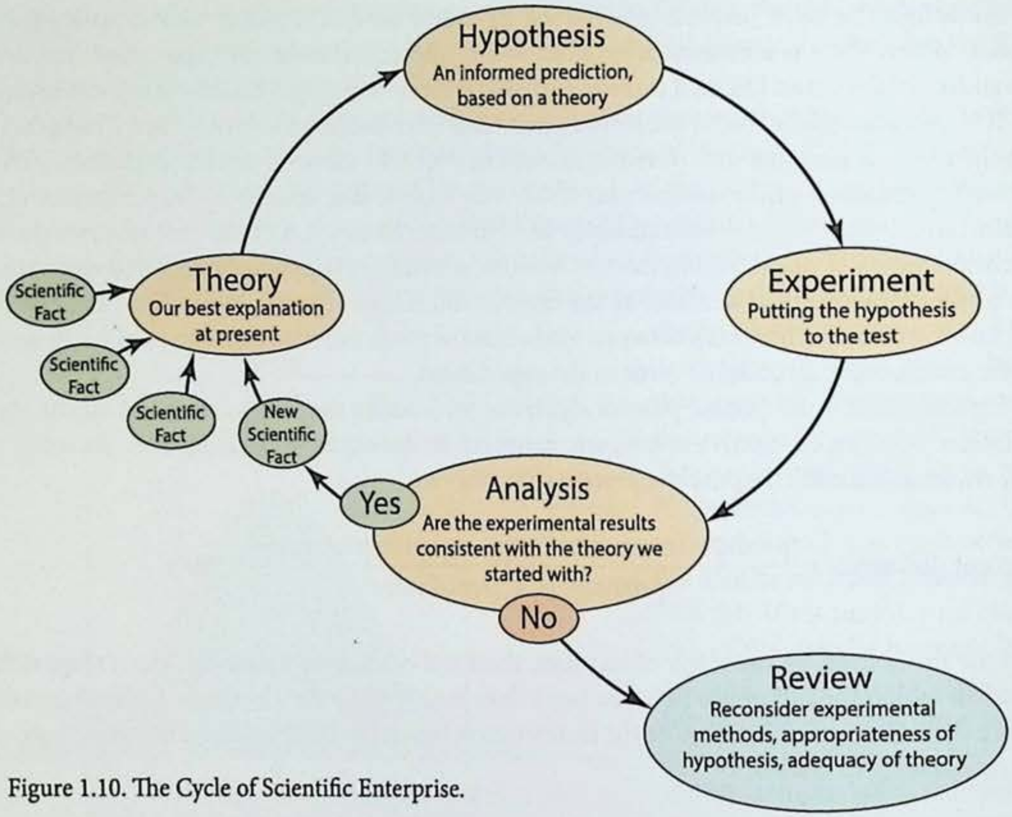


Figure 1.10. The Cycle of Scientific Enterprise.

science is not in the business of making truth claims. It is in the business of modeling how nature works with theories based on research.

As our theories develop over time, our hope is that they get closer and closer to the truth, the amazing and profound truth about mysteries such as what protons and electrons are, why they have the properties they have, and how the two most successful theories of the 20th century—quantum mechanics and general relativity—can be reconciled with each other. But the truth about nature is always out in front of us somewhere, always outside of our grasp. To know the truth about nature we would have to understand nature as God understands it. We are nowhere close to that.

For this reason, all scientific knowledge—facts and theories—is regarded as *provisional* and *corrigible*. Facts may be regarded as correct, and theories may be regarded as our best approximation to the reality of nature, but all such knowledge is subject to change and correction, to being replaced by more accurate facts or theories.

1.3.2 Truth and Facts

Christians believe in truths that have been revealed to us, that are absolute and unchanging. Scientific facts, by their very nature, are not like this, so definitions for *truth* and for *scientific facts* must take this into account. First we look at the ways that humans can know truth.

One way we know truth is when it is *evident* or *obvious* to us. For example, it is evident or obvious to you that you are awake right now while you are reading this text. Thus, it is correct to say that it is true you are awake. Likewise, you probably know if you have eaten a meal within the past three days. If so, then it is obvious to you that if you said, "I have eaten a meal within the past three days" you would be speaking the truth.

Knowing obvious truths is clearly an important part of living as a human being in God's world. God has made us so we can know truth about the world around us just by being here, seeing things, and remembering things. We rely on the stability of the world around us and our memories of it, because if it were otherwise we would probably all go crazy. The stability of the world and our memories are parts of the gift of creation God has given to us.

The second way humans can know truth is by the use of valid logic, beginning with true premises. I do not develop this idea here, since the use of formal logic has little direct relation to our work in this text. The exception is the use of mathematical logic to discover new mathematical truth. This is, of course, at the core of what mathematicians do, and also plays a role in problem solving.

The third way for us to know truth is for God to *reveal* it to us. Much truth is knowledge that is revealed to us by God, either by Special Revelation or by General Revelation. Special Revelation is the term theologians use to describe truths God teaches us in the Bible, his Holy Word. General Revelation refers to truths God teaches us through the world he made. Sometimes people describe Special and General Revelation as the two "books" of God's revelation to us, the book of God's *Word* (the Bible) and the book of God's *Works* (nature).

Since truth is obvious, or revealed by God (either through his word or his works), this means that truth is not discovered the same way scientific facts are discovered. Truth is true for all people, all times and all places. Truth is permanent and never changes. Some examples of revealed truths are:

- Jesus is the divine Son of God (Matthew 16:16).
- All have sinned and fall short of what God requires (Romans 3:23).
- God is the creator of all that is (Colossians 1:16, Revelation 4:11).
- God loves us (John 3:16).

Each of these statements is true, and we know they are true because God has revealed them to us in his word. (The reasons for believing God's word are important for all of us to know and understand, but that is a subject for a different course of study.) These truths are all unchanging, just like all truth. The distinction between truth and scientific facts, which I describe in the next section, is crucial. As believers, we embrace the absolute truths we find in Scripture. Scientific facts can change; truth does not. If we confuse these terms we may have a hard time discussing and defending the faith and distinguishing between the precious truths we know absolutely, such as that Jesus is our redeemer and rose from the dead, and things that may turn out not to be correct after all.

1.3.3 Scientific Facts, Theories, Hypotheses, and Experiments

To expound on the Cycle of Scientific Enterprise a bit further, the following are some definitions to keep in mind as you consider the mathematical models we develop in coming chapters.

Scientific Facts A scientific fact is a proposition based on a large amount of scientific data that is correct so far as we know. Scientific facts are discovered by experiment, observation, and inferences from experiments and observations. Scientific facts can and do change as new scientific knowledge—new data—is acquired. Since scientific facts are always subject to change, careful scientists will usually avoid terms such as *true* or *proven* to describe scientific facts. Instead, we say a scientific fact is correct so far as we know.

Scientific facts are just a small step away from raw data. To talk about what scientific facts mean, we must relate them together in a consistent explanatory framework. This is where theories come in.

Theories A theory is a mental model that accounts for the data (facts) in a certain field of research, and attempts to relate them, interpret them, and explain them. Scientific theories are successful if they repeatedly allow scientists to form new hypotheses that are supported by experiment. *Successful theories are the glory and goal of science.* Nevertheless, theories, like facts, are provisional and subject to change. Indeed, theories are almost constantly evolving as research continues. And as with scientific facts, when referring to theories, we avoid terms like *true* or *proven*. Instead, we speak in terms of how successful theories have been in generating hypotheses that are supported by experiments, that is, how accurately predictions derived from the theory match the results of experiments. A widely accepted scientific theory should be understood as our best explanation at present—our best model of how nature works.

It is important to realize that there is no scientific knowledge that is not theoretical. That is, data or facts by themselves don't tell us anything apart from the theories we have to account for and explain the scientific facts. For example, it is a scientific fact that radium is radioactive; a lump of it glows constantly from the radiation it emits. But so what? What does this bare fact mean and what is its significance? What is radiation and what causes it? Our present theoretical model associates this type of radiation with the atomic nucleus and the process of nuclear decay. Some nuclear structures are less stable than others, and larger atoms with unstable nuclei typically exhibit nuclear decay until the nucleus reaches the stable structure of the nucleus of lead, element 82. Each different radioactive element decays at a rate that is very predictable using a statistical mathematical model. With this theoretical basis, we understand the behavior of the atomic nucleus. The fact of nuclear decay is explained and understood in terms of nuclear theory.

It is also important to note that scientific theories need to be testable. A theory that does not lead to testable hypotheses has no chance of gaining credibility and remains at the level of conjecture. For a theory to be well established, it must lead to hypotheses that can be put to the test.

Hypotheses A hypothesis[1] is an informed prediction, with a justification, about what will happen in certain circumstances. Every hypothesis is based on a particular theory. It is hypotheses that are tested and thereby supported or not supported by scientific experiments. To form an experimental hypothesis, the scientist must understand the subject at issue according to a theoretical framework of some kind. This theoretical framework determines the hypotheses scientists form and test.

For an example, we can continue with the theory of nuclear decay. We have found that all the elements discovered so far with atomic numbers $Z = 84$ and higher are radioactive. Our theory of nuclear stability accounts for this. An obvious hypothesis we could form regarding other heavy elements not yet discovered—elements with atomic numbers $Z = 119$ and higher—is that when we are able to identify such an element, we expect atoms of the element to decay due to the instability of their nuclei.

Experiments An experiment is designed to test a particular hypothesis. If a hypothesis is supported by experimental results, and if other scientists are able to conduct experiments that also support the hypothesis, then the new scientific facts gained from the experimental results become additional support for the theory the hypothesis came from.

Continuing again with the theory of nuclear decay as an example, experiments in recent decades have been conducted repeatedly to identify the elements in the last period of the Periodic Table of the Elements. Since plutonium, $Z = 94$, is the heaviest naturally occurring element, scientists have expected that any element discovered with an atomic number greater than 94 will exhibit nuclear decay. And so far, experiments have repeatedly demonstrated this to be the case.

Sometimes, an experiment or series of experiments fails to support a particular hypothesis. When this occurs, it is not correct to say that the theory that led to the hypothesis is immediately disproved. In fact, there are many factors that could lead to such a lack of support. This is

1 A generation ago, correct usage required us to write *an hypothesis*, and some writers continue this usage today. I now use the more common contemporary usage *a hypothesis*.

why the Cycle of Scientific Enterprise includes the steps called *Analysis* and *Review*. The Review process essentially works in reverse around the Cycle of Scientific Enterprise to discover why the hypothesis is not supported by the data. The first step in the review process is to review the experiment: protocols, instruments, data collection methods, apparatus, lurking variables, and a host of other factors that could have led to the negative result. If the experimental data appear to be valid, the review process turns next to the hypothesis. Contemporary theories are mathematically complex and it is possible that the hypothesis is not properly formed from the theory. If the hypothesis checks out, the final step is to examine the theory itself. An unsupported hypothesis means the theory does not deliver a correct prediction, and such a failure calls the theory's integrity into question. No theory is perfect and no theory explains all relevant data. A failed prediction indicates that there is more to the picture than scientists know about. A long series of failed predictions wears down a theory's credibility, setting the stage for eventual replacement by a new theory that explains all the data more adequately than its predecessor. However, the complexity of scientific theories these days is such that producing a new theory can be very challenging.

Physics is a subject loaded with scientific facts and heavily based on theories—models—that we know are incomplete descriptions of nature. That is why the research continues, as our models (hopefully) get nearer and nearer to the truth.

1.4 Vector Methods

1.4.1 Scalars and Vectors

Scalar Quantities	Vector Quantities
temperature	velocity
mass	acceleration
energy	force
pressure	displacement
density	momentum

Table 1.6. Representative scalar and vector quantities.

When we model nature mathematically, we describe the quantities we seek to measure with variables and we develop equations that reflect our understanding of the relationships between the variables that are present in nature. One of the most fundamental aspects of describing quantities with variables is that simple quantities in nature may always be classified as one of two distinct types—*scalars* and *vectors*.[2] Representative examples of scalar and vector quantities are listed in Table 1.6. It is likely that the term *displacement* is unfamiliar. A displacement is a directed distance, or a distance along a certain direction. The discussion below clarifies the meaning of this term.

A scalar quantity can be expressed with a single value that indicates the size or amount of the quantity. Vector quantities cannot be expressed in terms of a single value. Vector quantities are inherently directional in character, and to express the value of a vector quantity both the *magnitude* and *direction* of the quantity must be stated.

For example, the temperature at a certain spot in a room can be expressed with a single value, such as 21°C, and thus temperature is a scalar quantity. By contrast, forces are directional and are thus vectors, so to describe a force completely the direction of the force must be stated along with its magnitude. The gravitational attraction of the earth on an object is an example of a force, and regardless where the object is, the direction of the force on the object is down (that is, toward the center of the earth). As another example, the force on a surface due to the pressure of a gas or liquid is perpendicular to the surface.

Two of the common vector quantities in physics, displacement and velocity, have common terms that refer to the magnitude of the vector (rather than both the magnitude and direction).

2 There is a third type—*tensors*—but tensor mathematics is very complex and does not typically appear in introductory or intermediate physics courses.

The magnitude of a displacement vector is called *distance* and the magnitude of a velocity vector is called *speed*. Since *distance* and *speed* refer specifically to the magnitudes of vectors, they refer to scalar quantities. If you say that a man walked 5 km, you have stated the distance the man walked—a scalar quantity. If you say that a man walked 5 km due northeast, you have stated the displacement the man underwent—a vector quantity. We address conventions for specifying a vector's direction in a moment, but I note here that there are a few common types of problems in physics that use geographical references to specify the direction of a vector, as I just did in this illustration.

Likewise, the term *speed* is a scalar quantity that refers to the magnitude of a velocity, a vector quantity. Thus, someone can refer to the speed of an electron as 276 m/s. But to state the velocity of the electron requires also stating its direction, which requires an angular reference of some kind. For solving problems in physics, the angular reference is often handled by representing the vector in question, the velocity of an electron in this case, as an arrow on a coordinate system. The magnitude of the vector is represented by the length of the arrow, and the direction of the vector is represented by the angle the arrow makes relative to the positive x-axis (or horizontal axis) in the coordinate system, as illustrated in Figure 1.11. As in trigonometry, positive angles are measured counterclockwise from the horizontal axis. With this in mind, one way of stating the velocity of the electron in the figure would be to use a symbol for the angle: 276 m/s, △ = 42°. Another way to state the velocity is to write it out: 276 m/s at an angle of 42° relative to the positive x-axis. A third and very common notation is to use variables—an italicized letter for the magnitude and an italicized Greek letter for the angle, as in $v = 276$ m/s, $\theta_v = 42°$.

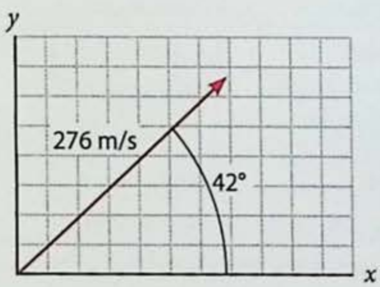


Figure 1.11. Specifying the direction of a vector quantity.

You are not necessarily required to use the positive x-axis for specifying angles, and there will be times when it is more convenient to use a different reference. Any convenient reference (such as the positive y-axis) may be used, so long as the reference is specified. As a convention in physics, directions are usually specified as relative to the positive x-axis. In the absence of any other specification, this is assumed.

The conventions for the notation used to denote vector quantities are summarized in Table 1.7. In print, different fonts are used to indicate scalar and vector quantities. An italic font is typically used for scalar quantities; a bold font is standard for vector quantities. In a text like this one, I indicate a distance of 5 km by typing $d = 5$ km. To indicate the displacement of 5 km NE, I type $\mathbf{d} = 5$ km, NE. The electron speed is printed as $v = 276$ m/s. Its velocity could be printed with an angle symbol as $\mathbf{v} = 276$ m/s, △ = 42°. However, not everyone recognizes the angle symbol, so it is more common (and formal) to use a variable for both the magnitude and direction, like this: $v = 276$ m/s, $\theta = 42°$. If necessary, subscripts are used on the angle variables to make it clear which vector the angle is associated with: $v = 276$ m/s, $\theta_v = 42°$. Another standard notation is to use the absolute value symbol to indicate a vector's magnitude, and thus write $|\mathbf{v}| = 276$ m/s, $\theta = 42°$.

	In Print	By Hand
Scalar	d, v	d, v
Vector	$\mathbf{d}$, $\mathbf{v}$	$\vec{d}$, $\vec{v}$
Magnitude of Vector	d, v, $\lvert\mathbf{d}\rvert$, $\lvert\mathbf{v}\rvert$	d, v, $\lvert\vec{d}\rvert$, $\lvert\vec{v}\rvert$
Angle of Vector	θ, △	θ, △

Table 1.7. Notation conventions for scalar and vector quantities.

Obviously, it is difficult to use italic and bold fonts when you are writing by hand on your exercises and exams. In that case, the convention is to place a little right-pointing arrow over a variable if you wish to indicate a vector, as shown in the table. Occasionally, printed materials use both bold print and an arrow over variable, such as $\vec{\mathbf{v}}$. When writing by hand, the most common way to indicate the magnitude of a vector is to use the absolute value symbol with the variable inside with an arrow over it: $|\vec{d}|$.

1.4.2 How to Learn Vector Addition

You already know how to solve problems using scalar quantities. You simply add, subtract, multiply, divide, and so on. When we add numbers this way we are adding *algebraically*. However, solving problems involving vector quantities requires using *vector arithmetic*. When we add two vector quantities together we must add them *vectorially*. Since a large percentage of problems in physics involve vectors, it is essential that you master the rules of vector arithmetic immediately. I encourage you to study this section carefully and work the exercises until you are certain you can execute vector calculations just as reliably and efficiently as you can execute scalar calculations.

When solving vector problems in physics, graphical aids are universally used to set the problem up and put together a solution strategy. This occurs over and over in this text and you should draw similar graphical depictions to aid you in your solutions. As I explain above, vector quantities are represented graphically as arrows on a coordinate system. The length of the arrow represents the magnitude of the vector and the direction of the vector is indicated by the angle it makes with a reference line (typically, the positive x-axis).

We first consider vector addition (Sections 1.4.3 through 1.4.5). There are two ways to execute a calculation involving vector addition. One is to perform the calculation graphically, using a rule and protractor. This is a time-consuming method and the accuracy of your results depends entirely on the accuracy of your drafting technique. The second method is to use trigonometry to work out an analytical solution. This method delivers an accurate answer (so long as you do it correctly) and is much faster.

You might be thinking that we should dispense with the graphical approach and get straight to the trigonometric approach. However, decades of experience teaching high school physics has convinced me that you should learn the graphical method first and solve a few problems with it so that you learn to visualize what you are doing. After that, you can learn the trigonometric approach and use it exclusively from that point onwards. I have seen many occasions when students tried to jump into the trigonometric methods of vector arithmetic without learning the graphical approach first. Most of the time this results in students having very little understanding of what they are doing, and this leads to disaster when we begin solving problems using vector methods. The moral of this story is that you need to spend a day with your rule and protractor getting the hang of this, even though it may seem like you're back in geometry class. A couple of hours of practice will pay off and help ensure that you quickly and effectively learn the trigonometric methods to follow.

After we address vector addition in the next three sections, we address vector multiplication in Sections 1.4.6 and 1.4.7. You do not need to know the methods presented in Section 1.4.7 until we get to the chapters on Static Equilibrium (Chapter 4) and Energy (Chapter 5). But I include the material in this chapter with the other material on vector arithmetic to make it easy to find later. For now, study Sections 1.4.3 through 1.4.5 carefully and do the problems in the exercises. Then proceed to Chapter 2. When the time comes that you need to know about vector multiplication, I will send you back to Section 1.4.7 to learn about it.

1.4.3 Vector Addition—the Graphical Method

Two vectors, **A** and **B**, are shown in Figure 1.12. Each has a tip and a tail, as indicated. The angle a vector makes is measured at the tail of the vector, as if the tail of the vector were placed at the origin of the coordinate system. The lengths of the vectors are drawn proportionally, using an appropriate scale, such as 1/4 inch = 1 unit, 1 inch = 10 units, etc. In the figure, vector **A** has a magnitude of 6.2 units and is at an angle of 42° relative to the positive x-axis. (Just imagine the x-axis beginning at the vector's tail and pointing to the right. It is not necessary actually to draw an axis.) Vector **B** has a magnitude of 9.5 units and is at an angle of −15° relative to the positive x-axis. Note that in the diagram, the angle of **B** is labeled with a positive value. The geometric value of the angle between those two line segments is 15°. However, the *direction* of the vector must be stated as −15°, because the 15° angle the vector makes with the positive x-axis is in the clockwise direction, not the counterclockwise direction.

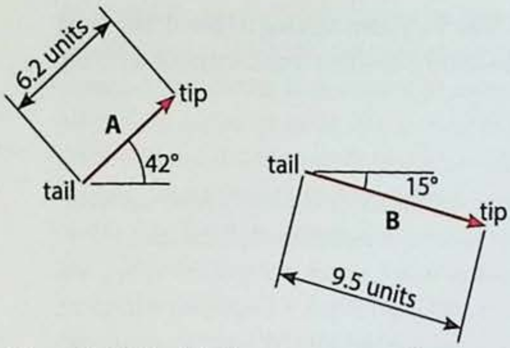


Figure 1.12. Graphical representations of two vectors, **A** and **B**.

As with scalar addition, vector addition is commutative—several vectors may be added in any order and the result is the same. The result of a vector addition is called the *resultant vector*, or simply the *resultant*. The resultant is a new vector, with its own magnitude and direction.

Graphical vector addition is illustrated in Figure 1.13. To add vectors **A** and **B**, place the tail of the second vector at the tip of the first vector. (Again, the operation is commutative; it doesn't matter which is first or second.) Preserve the vectors' lengths and angles while doing this. The resultant vector, **R**, is the vector whose tail is at the tail of the first vector and whose tip is at the tip of the last vector. In other words, draw **R** so that it points from the tail of the first vector to the tip of the last vector. Then measure the length of **R** with a rule to get the magnitude of **R**, denoted by $|\mathbf{R}|$ or R, and use a protractor to measure the angle **R** makes with the positive x-axis, denoted by θ_R. Drawing these vectors and making the measurements indicates that $|\mathbf{R}|$ = 13.9 units, and $\theta_R = 7.0°$.

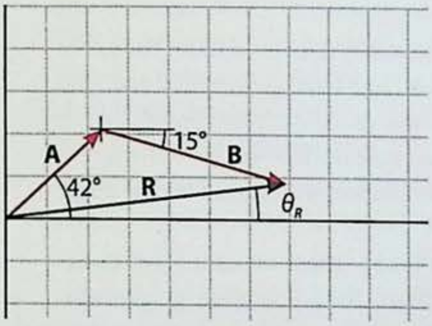


Figure 1.13. Graphical addition of vectors **A** and **B** to produce resultant vector, **R**.

In Figure 1.14, the same two vectors are depicted but the order of the addition is reversed—vector **A** is added to **B**, instead of **B** being added to **A**. This diagram demonstrates the commutativity of vector addition, since the resultant **R** is the same vector as before.

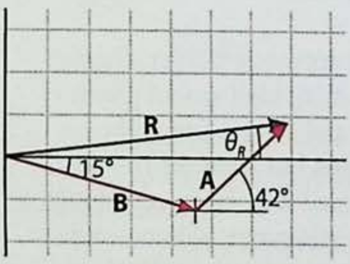


Figure 1.14. Adding vector **A** to vector **B** produces the same resultant as adding **B** to **A**.

In Figures 1.13 and 1.14, the vectors are added by drawing them tip to tail. However, Figure 1.15 demonstrates one additional feature of vector addition. If both vectors are drawn with their tails at the origin, they form adjacent sides of a parallelogram. Completing the other two sides, you can see that the resultant **R** of adding **A** + **B** is the diagonal of the parallelogram. Be careful here though: drawing a vector along the other diagonal, from the tip of **A** to the tip of **B**, would represent an *entirely different* addition problem: **A** + **R** = **B**. This is not the same as **A** + **B** = **R**. Take care to avoid this confusion.

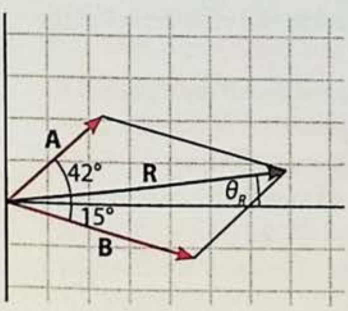


Figure 1.15. When both vectors **A** and **B** are drawn at the origin, they form adjacent sides of a parallelogram. The diagonal of the parallelogram is the resultant vector, **R**.

When adding vectors graphically, draw the vectors with a sharp pencil on graph paper, using as large a scale as possible. The lines on the graph paper assist you in orienting your protractor and a large

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