Solutions Manual

Saxon Calculus

with Trigonometry and Analytic Geometry

SECOND EDITION

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Problem Set 1

1. A. $7\frac{1}{4}\text{ ft}^2 = 7.25\text{ ft}^2$

B. $0.8\text{ yd}^2 \times \dfrac{3^2\text{ ft}^2}{1\text{ yd}^2} = 7.2\text{ ft}^2$

Quantity A is greater: **A**

2. A. $7(2t - 2t) = 7(0) = 0$

B. $-6(3t - 3t) = -6(0) = 0$

Quantities A and B are equal: **C**

3. If $x = 8$ and $y = 3$, then quantity A is greater.

If $x = 5$ and $y = 13$, then quantity B is greater.

If $x = 6$ and $y = 6$, then the quantities are equal.

Insufficient information: **D**

4. $a = \dfrac{3 + 6}{2} = 4.5$

A. $3a = 13.5$

B. $a + 6 = 10.5$

Quantity A is greater: **A**

5.
$$\frac{m}{x} = y\left(\frac{1}{R_1} + \frac{a}{R_2}\right)$$
$$\frac{m}{x} = \frac{y}{R_1} + \frac{ay}{R_2}$$
$$mR_1R_2 = R_2xy + R_1axy$$
$$mR_1R_2 - R_1axy = R_2xy$$
$$R_1(mR_2 - axy) = R_2xy$$
$$R_1 = \frac{R_2xy}{mR_2 - axy}$$

6. $a + \dfrac{1}{a + \frac{1}{a}} = a + \dfrac{1}{\frac{a^2 + 1}{a}} = a + \dfrac{a}{a^2 + 1}$

$$= \frac{a^3 + 2a}{a^2 + 1}$$

7. $\dfrac{1}{a + \frac{1}{x + \frac{1}{m}}} = \dfrac{1}{a + \frac{1}{\frac{mx + 1}{m}}} = \dfrac{1}{a + \frac{m}{mx + 1}}$

$$= \frac{1}{\frac{amx + a + m}{mx + 1}} = \frac{mx + 1}{amx + a + m}$$

8. $\dfrac{x^2y}{1 + m^2} + \dfrac{x}{y} = \dfrac{x^2y^2}{y(1 + m^2)} + \dfrac{x(1 + m^2)}{y(1 + m^2)}$

$$= \frac{x^2y^2 + x + m^2x}{y + m^2y}$$

9. $\dfrac{4 - 3\sqrt{2}}{8 - \sqrt{2}} = \dfrac{4 - 3\sqrt{2}}{8 - \sqrt{2}}\left(\dfrac{8 + \sqrt{2}}{8 + \sqrt{2}}\right)$

$$= \frac{32 + 4\sqrt{2} - 24\sqrt{2} - 6}{64 - 2} = \frac{26 - 20\sqrt{2}}{62}$$

$$= \frac{13 - 10\sqrt{2}}{31}$$

10. $\dfrac{x^a y^{a+b}}{x^{-a/2}y^{b-1}} = x^a x^{a/2} y^{a+b} y^{-b+1} = x^{3a/2}y^{a+1}$

11. $\dfrac{m^{x+2}b^{x-2}}{m^{2x/3}b^{-3x/2}} = m^{x+2}m^{-2x/3}b^{x-2}b^{3x/2}$

$$= m^{x/3+2}b^{5x/2-2}$$

12. $\sqrt{xy}\,x^{2/3}y^{-3/2} = x^{1/2}x^{2/3}y^{1/2}y^{-3/2} = x^{7/6}y^{-1}$

13.
$$\begin{cases} 2x + 3y = -4 \\ x - 2z = -3 \\ 2y - z = -6 \end{cases}$$
$$z = 2y + 6$$
$$x - 2(2y + 6) = -3$$
$$x - 4y = 9$$
$$x = 4y + 9$$
$$2(4y + 9) + 3y = -4$$
$$8y + 18 + 3y = -4$$
$$11y = -22$$
$$y = -2$$
$$x = 4(-2) + 9 = 1$$
$$z = 2(-2) + 6 = 2$$
$$(1, -2, 2)$$

14. $a^2x - a^2 - 4b^2x + 4b^2$

$$= a^2(x - 1) - 4b^2(x - 1)$$
$$= (a^2 - 4b^2)(x - 1)$$
$$= (a - 2b)(a + 2b)(x - 1)$$

15. $16a^{4m+3} - 8a^{2m+3} = 8a^{2m+3}(2a^{2m} - 1)$

16. $a^2b^{2x+2} - ab^{2x+1} = ab^{2x+1}(ab - 1)$

17. $9x^2 - y^4 = (3x)^2 - (y^2)^2 = (3x + y^2)(3x - y^2)$

18. $a^6 - 27b^3c^3 = (a^2)^3 - (3bc)^3$
$= (a^2 - 3bc)(a^4 + 3a^2bc + 9b^2c^2)$

19. $x^3y^6 + 8m^{12} = (xy^2)^3 + (2m^4)^3$
$= (xy^2 + 2m^4)(x^2y^4 - 2m^4xy^2 + 4m^8)$

20. $\dfrac{12!}{8!\,4!} = \dfrac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8!}{8!\,4!} = \dfrac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2}$
$= 11 \cdot 5 \cdot 9 = \mathbf{495}$

21. $\dfrac{n \cdot (n!)}{(n+1)!} = \dfrac{n \cdot n!}{(n+1) \cdot n!} = \dfrac{n}{n+1}$

22. $\displaystyle\sum_{i=1}^{3} 4 = 4 + 4 + 4 = \mathbf{12}$

23. $\displaystyle\sum_{m=0}^{3} \frac{3^m}{m+1} = 1 + \frac{3}{2} + 3 + \frac{27}{4} = \frac{\mathbf{49}}{\mathbf{4}}$

24. $V = \dfrac{4}{3}\pi r^3$

$\dfrac{4}{3}\pi = \dfrac{4}{3}\pi r^3$

$r = 1$

$A = 4\pi r^2$
$= 4\pi(1)^2$
$= \mathbf{4\pi\ m^2}$

25. $A = \pi r^2$
$4\pi = \pi r^2$
$r = 2$

$V = \dfrac{1}{3}\pi r^2 h = \dfrac{1}{3}\pi\, 2^2(4)$

$= \dfrac{\mathbf{16}}{\mathbf{3}}\pi\ \mathbf{cm^3}$

Problem Set 2

1. Midpoint $= \left(\dfrac{4+10}{2}, \dfrac{2-2}{2}\right) = (7, 0)$

$d = \sqrt{(7-6)^2 + (0-8)^2}$
$= \sqrt{1 + 64} = \sqrt{\mathbf{65}}$

2. $(5\sqrt{2})^2 = y^2 + 5^2$
$50 = y^2 + 25$
$y^2 = 25$
$y = \mathbf{5}$

3. $2x - 3y + 2 = 0$
$3y = 2x + 2$
$y = \dfrac{\mathbf{2}}{\mathbf{3}}x + \dfrac{\mathbf{2}}{\mathbf{3}}$

4. $4y = -3x + 2$
$y = -\dfrac{3}{4}x + \dfrac{1}{2}$

slope $= -\dfrac{3}{4}$ $\quad \perp$slope $= \dfrac{4}{3}$

$y + 1 = \dfrac{4}{3}(x - 1)$
$y = \dfrac{\mathbf{4}}{\mathbf{3}}x - \dfrac{\mathbf{7}}{\mathbf{3}}$

5. $x^2 - 3x - 4 = 0$
$x^2 - 3x = 4$
$x^2 - 3x + \dfrac{9}{4} = 4 + \dfrac{9}{4}$
$\left(x - \dfrac{3}{2}\right)^2 = \dfrac{25}{4}$
$x = \dfrac{3}{2} \pm \dfrac{5}{2}$
$x = \mathbf{4, -1}$

6. $2x^2 = x + 3$
$2x^2 - x = 3$
$x^2 - \dfrac{1}{2}x = \dfrac{3}{2}$
$x^2 - \dfrac{1}{2}x + \dfrac{1}{16} = \dfrac{3}{2} + \dfrac{1}{16}$
$\left(x - \dfrac{1}{4}\right)^2 = \dfrac{25}{16}$
$x = \dfrac{1}{4} \pm \dfrac{5}{4}$
$x = \dfrac{\mathbf{3}}{\mathbf{2}}, \mathbf{-1}$

7. $3x^2 - x - 7 = 0$
$x = \dfrac{1 \pm \sqrt{1 - 4(3)(-7)}}{6}$
$x = \dfrac{\mathbf{1}}{\mathbf{6}} \pm \dfrac{\sqrt{\mathbf{85}}}{\mathbf{6}}$

8.
$$\begin{array}{r} 2x^2 + 6x + 15 \\ x - 3 \overline{)\,2x^3 + 0x^2 - 3x + 5} \\ \underline{2x^3 - 6x^2} \\ 6x^2 - 3x \\ \underline{6x^2 - 18x} \\ 15x + 5 \\ \underline{15x - 45} \\ 50 \end{array}$$

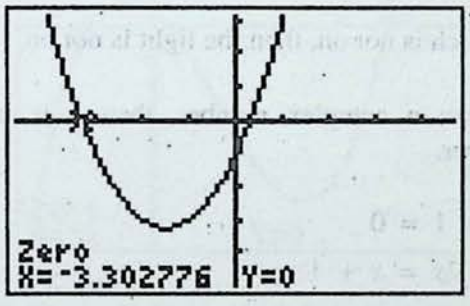
$$2x^2 + 6x + 15 + \frac{50}{x - 3}$$

9.
$$\begin{cases} xy = -4 \\ y = -x - 2 \end{cases}$$

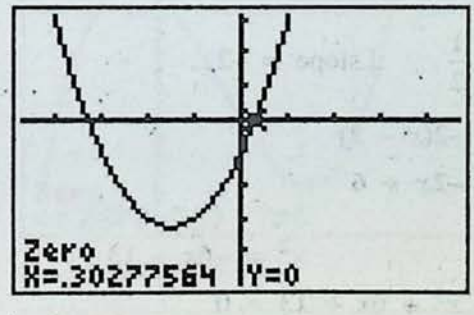
$$x(-x - 2) = -4$$
$$-x^2 - 2x = -4$$
$$x^2 + 2x - 4 = 0$$
$$x = \frac{-2 \pm \sqrt{4 - 4(1)(-4)}}{2}$$
$$x = -1 \pm \sqrt{5}$$

$(-1 + \sqrt{5}, -1 - \sqrt{5}), (-1 - \sqrt{5}, -1 + \sqrt{5})$

10.

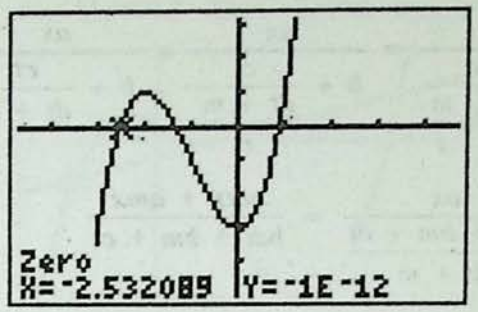


$x \approx -3.302776$

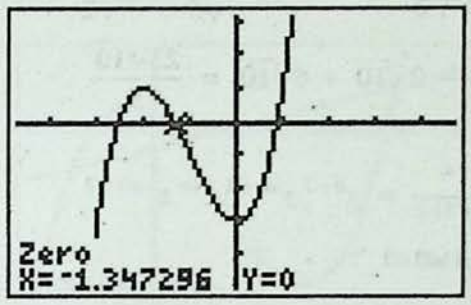


$x \approx 0.30277564$

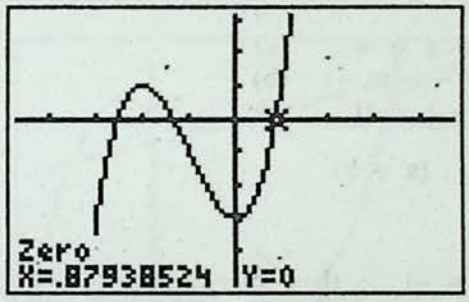
11.



$x \approx -2.532089$

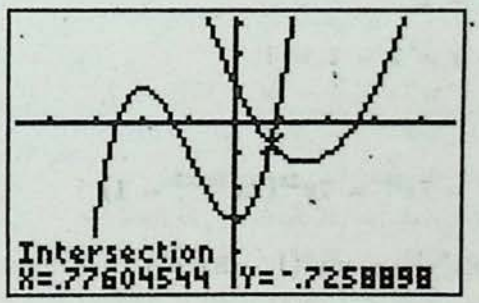


$x \approx -1.347296$



$x \approx 0.87938524$

12. Let $Y_1 = X^2 - 3X + 1$ and $Y_2 = X^3 + 3X^2 - 3$.



(0.77604544, –0.7258898)

13.
$$k^2 = \frac{1}{bc}\left(\frac{x}{3} - \frac{6y}{d}\right)$$
$$k^2 = \frac{x}{3bc} - \frac{6y}{bcd}$$
$$3bcdk^2 = dx - 18y$$
$$dx = 3bcdk^2 + 18y$$
$$x = \frac{3bcdk^2 + 18y}{d}$$

14. $$\frac{ax}{b+\dfrac{c}{d+\dfrac{m}{t}}} = \frac{ax}{b+\dfrac{c}{\dfrac{dt+m}{t}}} = \frac{ax}{b+\dfrac{ct}{dt+m}}$$

$$= \frac{ax}{\dfrac{bdt+bm+ct}{dt+m}} = \frac{adtx+amx}{bdt+bm+ct}$$

15. $$3\sqrt{\frac{2}{5}} - 4\sqrt{\frac{5}{2}} + 3\sqrt{40} = \frac{3\sqrt{2}}{\sqrt{5}} - \frac{4\sqrt{5}}{\sqrt{2}} + 6\sqrt{10}$$

$$= \frac{3\sqrt{10}}{5} - 2\sqrt{10} + 6\sqrt{10} = \frac{23\sqrt{10}}{5}$$

16. $$\frac{y^{a-2}z^{4a}}{y^{-2a-1}z^{a/3+2}} = y^{a-2}y^{2a+1}z^{4a}z^{-a/3-2}$$

$$= y^{3a-1}z^{11a/3-2}$$

17. $$\sqrt{x^3y^3}\,y^{1/3}x^{2/3} = x^{3/2}x^{2/3}y^{3/2}y^{1/3} = x^{13/6}y^{11/6}$$

18.
$$\begin{cases} x+y+z=4 & \text{(a)} \\ 2x-y-z=-1 & \text{(b)} \\ x-y+z=0 & \text{(c)} \end{cases}$$

$3x = 3$ (a + b)

$x = 1$

$3x - 2y = -1$ (b + c)

$3 - 2y = -1$

$-2y = -4$

$y = 2$

$z = y - x = 2 - 1 = 1$

(1, 2, 1)

19. $14x^{4b-2} - 7x^{2b} = 7x^{2b}(2x^{2b-2} - 1)$

20. $x^3y^6 - 8x^6y^{12} = x^3y^6(1 - 8x^3y^6)$

$= x^3y^6[(1)^3 - (2xy^2)^3]$

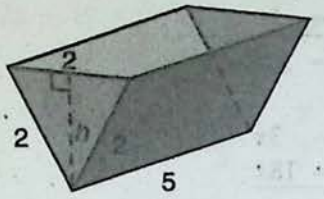
$= x^3y^6(1 - 2xy^2)(1 + 2xy^2 + 4x^2y^4)$

21. $$\frac{n!}{(n-1)!} = \frac{n\cdot(n-1)!}{(n-1)!} = n$$

22. $$\sum_{n=1}^{3}(n^2-2) = -1 + 2 + 7 = 8$$

23. $$\sum_{j=-2}^{1}\frac{2j-3}{3} = -\frac{7}{3} - \frac{5}{3} - 1 - \frac{1}{3} = -\frac{16}{3}$$

24.



$h = \sqrt{3}$

$V = (\text{Area}_{\text{Triangle}})(\text{Length})$

$= \frac{1}{2}(2)(\sqrt{3})(5)$

$= 5\sqrt{3}\ \text{m}^3 \approx 8.6603\ \text{m}^3$

25. If $x^2 = y^2$, then $x = \pm y$.

Insufficient information: **D**

Problem Set 3

1. If the switch is on, then the light is on.

2. If the light is not on, then the switch is not on.

3. If the switch is not on, then the light is not on.

4. If x is not a complex number, then x is not a real number.

5. $2y - x - 1 = 0$

$2y = x + 1$

$y = \frac{1}{2}x + \frac{1}{2}$

slope $= \frac{1}{2}$ $\perp$slope $= -2$

$y - 2 = -2(x - 2)$

$y = -2x + 6$

6. $x^2 = -6x - 13$

$x^2 + 6x + 13 = 0$

$x^2 + 6x + 9 + 13 - 9 = 0$

$(x + 3)^2 + 4 = 0$

7. $x^2 - 3x - 7 = 0$

$$x = \frac{3 \pm \sqrt{9 - 4(1)(-7)}}{2}$$

$$= \frac{3}{2} \pm \frac{\sqrt{37}}{2}$$

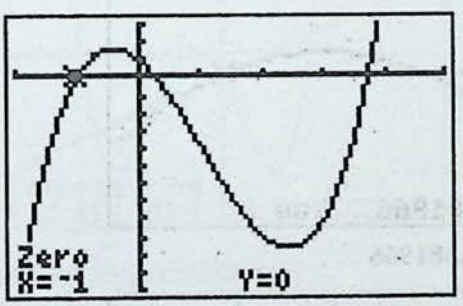
8. $\begin{cases} 2y^2 - x^2 = 1 \\ y + 1 = x \end{cases}$

$$2y^2 - (y+1)^2 = 1$$
$$2y^2 - y^2 - 2y - 1 = 1$$
$$y^2 - 2y - 2 = 0$$
$$y = \frac{2 \pm \sqrt{4 - 4(1)(-2)}}{2}$$
$$y = 1 \pm \sqrt{3}$$
$$x = y + 1 = 2 \pm \sqrt{3}y$$
$$(2 + \sqrt{3}, 1 + \sqrt{3}), (2 - \sqrt{3}, 1 - \sqrt{3})$$

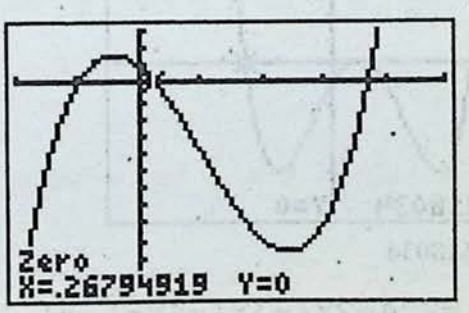
9.
$$\begin{array}{r} x^2 - 12x - 2 \\ x - 1 \overline{)\, x^3 - 13x^2 + 10x - 8} \\ \underline{x^3 - x^2} \\ -12x^2 + 10x \\ \underline{-12x^2 + 12x} \\ -2x - 8 \\ \underline{-2x + 2} \\ -10 \end{array}$$

$$x^2 - 12x - 2 - \frac{10}{x - 1}$$

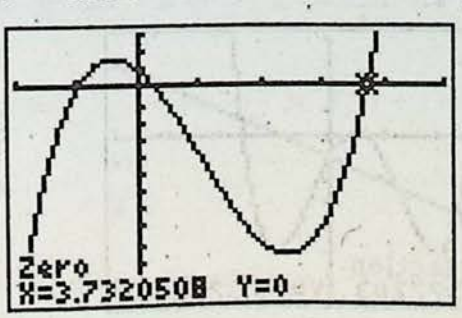
10.



$x = -1$

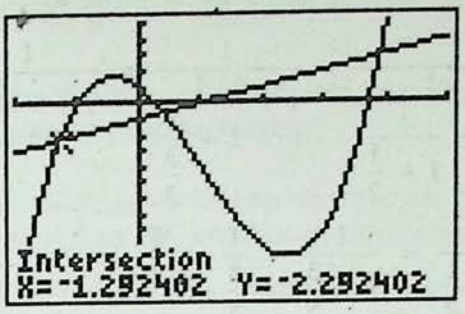


$x \approx 0.26794919$

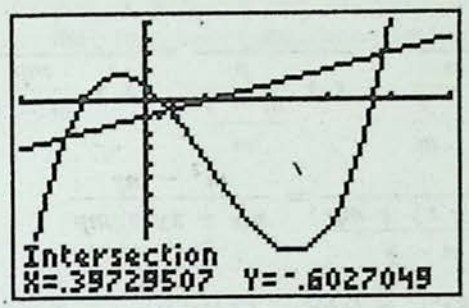


$x \approx 3.7320508$

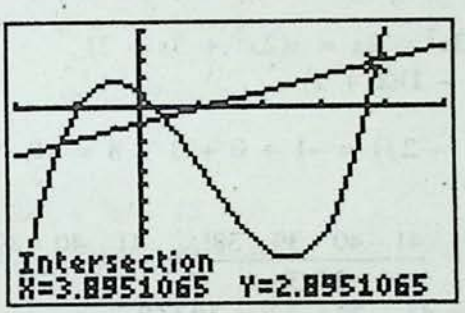
11. Let $Y_1 = X^3 - 3X^2 - 3X + 1$ and $Y_2 = X - 1$.



(–1.292402, –2.292402)



(0.39729507, –0.6027049)



(3.8951065, 2.8951065)

12.
$$\frac{m+b}{c} = \frac{1}{k}\left(\frac{a}{R_1} + \frac{b}{R_2}\right)$$
$$\frac{m+b}{c} = \frac{a}{kR_1} + \frac{b}{kR_2}$$
$$kR_1R_2(m+b) = acR_2 + bcR_1$$
$$kmR_1R_2 + bkR_1R_2 = acR_2 + bcR_1$$
$$kmR_1R_2 + bkR_1R_2 - bcR_1 = acR_2$$
$$R_1(kmR_2 + bkR_2 - bc) = acR_2$$
$$\frac{acR_2}{kmR_2 + bkR_2 - bc} = R_1$$

13. $\dfrac{4 - 2\sqrt{3}}{2 - \sqrt{3}} = \dfrac{2(2 - \sqrt{3})}{2 - \sqrt{3}} = 2$

14. $5\sqrt{\dfrac{3}{7}} - 2\sqrt{\dfrac{7}{3}} + \sqrt{84} = \dfrac{5\sqrt{21}}{7} - \dfrac{2\sqrt{21}}{3} + 2\sqrt{21}$

$$= \frac{43\sqrt{21}}{21}$$

15. $\sqrt{x^3y^5}\ y^{1/4}x^{3/2} = x^{3/2}x^{3/2}y^{5/2}y^{1/4} = x^3y^{11/4}$

16. $$\frac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{2}}}} = \frac{1}{1+\dfrac{1}{1+\dfrac{1}{\frac{3}{2}}}} = \frac{1}{1+\dfrac{1}{1+\frac{2}{3}}}$$
$$= \frac{1}{1+\dfrac{1}{\frac{5}{3}}} = \frac{1}{1+\frac{3}{5}} = \frac{1}{\frac{8}{5}} = \frac{5}{8}$$

17. $$\frac{m}{x+\dfrac{p}{1-\frac{y}{m}}} = \frac{m}{x+\dfrac{p}{\frac{m-y}{m}}} = \frac{m}{x+\frac{mp}{m-y}}$$
$$= \frac{m}{\frac{mx-xy+mp}{m-y}} = \frac{m^2-my}{mx-xy+mp}$$

18. $a^3b^3 - 8x^6y^9 = (ab)^3 - (2x^2y^3)^3$
$= (ab - 2x^2y^3)(a^2b^2 + 2abx^2y^3 + 4x^4y^6)$

19. $2x^3 + 3x^2 - 2x = x(2x^2 + 3x - 2)$
$= x(2x - 1)(x + 2)$

20. $\displaystyle\sum_{j=1}^{4}(j^2 - 2j) = -1 + 0 + 3 + 8 = \mathbf{10}$

21. $$\frac{41!}{38!\,3!} = \frac{41\cdot 40\cdot 39\cdot 38!}{38!\,3!} = \frac{41\cdot 40\cdot 39}{3\cdot 2}$$
$$= 41\cdot 20\cdot 13 = \mathbf{10{,}660}$$

22. $$\frac{a^2-b^2}{a+b} = \frac{(a+b)(a-b)}{(a+b)} = a - b$$

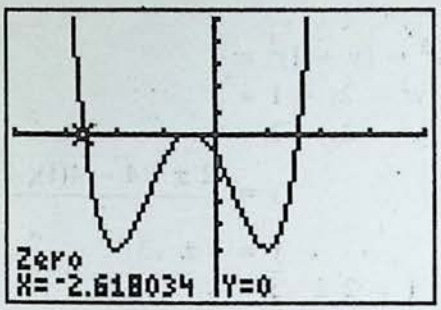
23. $$\frac{n!\,(n+1)!}{(n+2)!} = \frac{n!\,(n+1)!}{(n+2)(n+1)!} = \frac{n!}{n+2}$$

24. $$\frac{r}{h} = \frac{r}{h}$$
$$\frac{2}{6} = \frac{r}{2}$$
$$r = \frac{2(2)}{6} = \frac{2}{3}$$
$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{2}{3}\right)^2(2)$$
$$= \frac{8}{27}\pi\ \text{cm}^3 \approx \mathbf{0.9308\ cm^3}$$

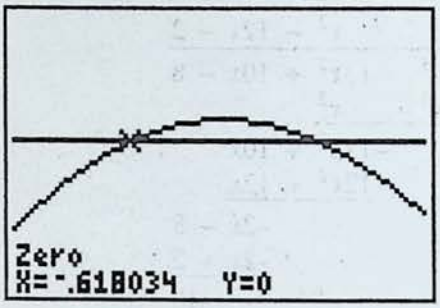
25. If x and y are both positive or both negative and $x > y$, then $\frac{1}{y} > \frac{1}{x}$. If x is positive and y is negative, then $\frac{1}{x} > \frac{1}{y}$.

Insufficient information: **D**

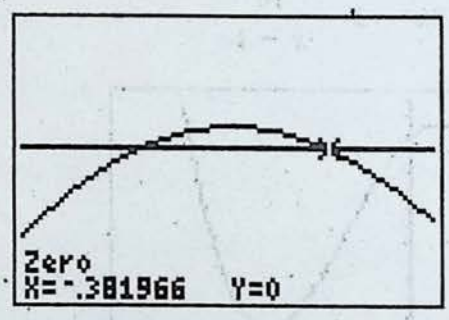
PROBLEM SET 4

1. 

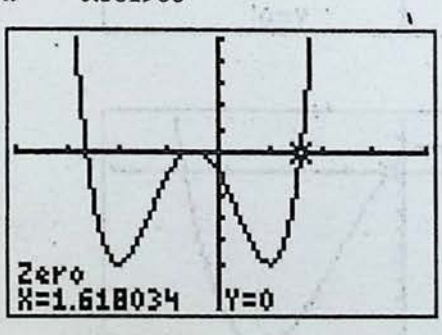
$x \approx -2.618034$



$x \approx -0.618034$

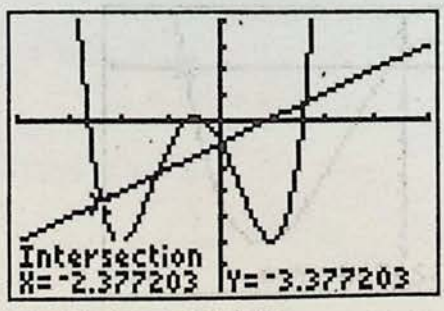


$x \approx -0.381966$

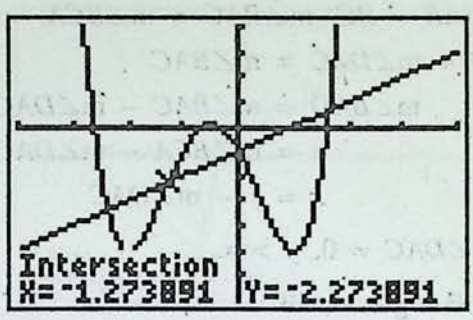


$x \approx 1.618034$

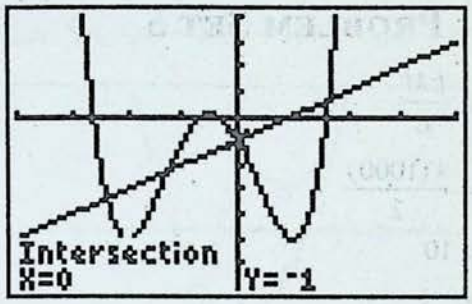
2. Let $Y_1 = X^4 + 2X^3 - 3X^2 - 4X - 1$ and $Y_2 = X - 1$.



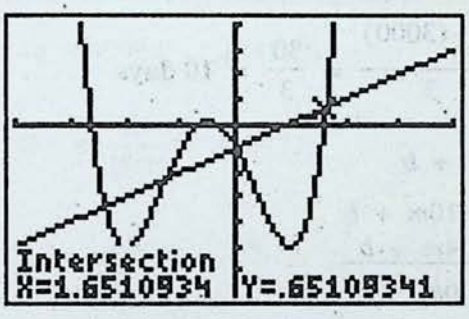
(–2.377203, –3.377203)



(–1.273891, –2.273891)



(0, –1)



(1.6510934, 0.65109341)

3. $\cos^2 \frac{\pi}{3} - \cot \frac{\pi}{4} + \sin \frac{\pi}{6} = \left(\frac{1}{2}\right)^2 - 1 + \frac{1}{2}$

$= -\frac{1}{4}$

4. $\sec 60° + \csc^2 \frac{\pi}{3} = 2 + \left(\frac{2}{\sqrt{3}}\right)^2 = \frac{10}{3}$

5. $3 \cos \frac{17\pi}{6} + 2 \cos -\frac{5\pi}{3} = 3 \cos \frac{5\pi}{6} + 2 \cos \frac{\pi}{3}$

$= 3\left(-\frac{\sqrt{3}}{2}\right) + 2\left(\frac{1}{2}\right) = -\frac{3\sqrt{3}}{2} + 1$

6. $4 \tan -\frac{3\pi}{4} + \sin -\frac{\pi}{4} = 4(1) + \left(-\frac{\sqrt{2}}{2}\right)$

$= 4 - \frac{\sqrt{2}}{2}$

7. $(\sin^2 \theta)(\csc \theta)(\cot \theta) = \left(\frac{\sin^2 \theta}{1}\right)\left(\frac{1}{\sin \theta}\right)\left(\frac{\cos \theta}{\sin \theta}\right)$

$= \cos \theta$

8. $\frac{\tan \theta \sin \theta}{\sec \theta} = \frac{\sin \theta}{\cos \theta} \sin \theta \cos \theta = \sin^2 \theta$

9. If a function is one-to-one, then it is not both increasing and decreasing.

10. Contrapositive: If your thumb does not hurt, then you did not hit your thumb with a hammer.

Converse: If your thumb hurts, then you hit your thumb with a hammer.

Inverse: If you did not hit your thumb with a hammer, then your thumb does not hurt.

11. $-3y = \frac{x}{3} + 2$

$y = -\frac{1}{9}x - \frac{2}{3}$

slope $= -\frac{1}{9}$

$y + 3 = -\frac{1}{9}(x + 9)$

$-9y - 27 = x + 9$

$x + 9y + 36 = 0$

12. $2x^2 + 7x - 15 = 0$

$(2x - 3)(x + 5) = 0$

$x = \frac{3}{2}, -5$

13. $x^2 + x - 1 = 0$

$x = \frac{-1 \pm \sqrt{1 - 4(1)(-1)}}{2}$

$x = -\frac{1}{2} \pm \frac{\sqrt{5}}{2}$

14. $(3x^2 - 4x + 5)(2x - 1)$

$= 6x^3 - 3x^2 - 8x^2 + 4x + 10x - 5$

$= 6x^3 - 11x^2 + 14x - 5$

15. $\begin{cases} x^2 + y^2 = 8 \\ x + y = 0 \end{cases}$

$y = -x$

$x^2 + (-x)^2 = 8$

$2x^2 = 8$

$x = \pm 2$

(2, –2), (–2, 2)

16. $\frac{1}{r} = v\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

$\frac{1}{r} = \frac{v}{r_1} + \frac{v}{r_2}$

$r_1 r_2 = r(r_2 v + r_1 v)$

$r = \frac{r_1 r_2}{v(r_1 + r_2)}$

17. $\frac{(n-1)!\,n!}{(n-2)!} = \frac{(n-1)(n-2)!\,n!}{(n-2)!} = (n-1)n!$

18. $5\sqrt{\frac{1}{5}} - 3\sqrt{5} + \sqrt{50} = \sqrt{5} - 3\sqrt{5} + 5\sqrt{2}$

$= \mathbf{5\sqrt{2} - 2\sqrt{5}}$

19. $\frac{x^3 - y^3}{x^2 + xy + y^2} = \frac{(x-y)(x^2 + xy + y^2)}{x^2 + xy + y^2}$

$= x - y$

20. $\sum_{i=-1}^{1} (2^i + i) = -\frac{1}{2} + 1 + 3 = \frac{7}{2}$

21. $\frac{1}{1 + \frac{1}{1 + \frac{1}{3}}} = \frac{1}{1 + \frac{1}{\frac{4}{3}}} = \frac{1}{1 + \frac{3}{4}} = \frac{1}{\frac{7}{4}} = \frac{4}{7}$

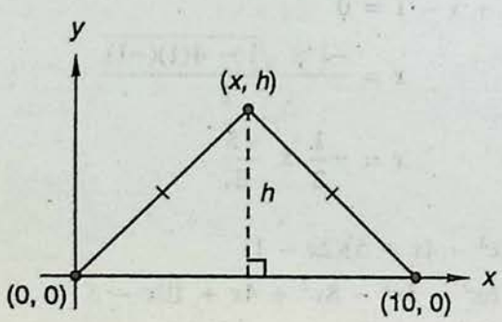
22. $SA = \text{Perimeter}_{\text{Base}} \cdot \text{Height} + 2\text{Area}_{\text{Base}}$

$= 2(w + l)(h) + 2(wl)$

$= 2(hw + hl) + 2(wl)$

$= \mathbf{2(hw + hl + lw)\ units^2}$

23.



$h = \frac{10}{2} = 5$

(5, 5)

24. If $0 < x < 1$, then $x > x^{10}$ and quantity A is greater.

If $x = 1$, then $x = x^{10}$ so A and B are equal.

If $x > 1$, then $x < x^{10}$ and B is greater.

Insufficient information: **D**

25. Because $AB = BC$, $m\angle BAC = m\angle BCA$.

$m\angle BAD + m\angle DAC = m\angle BAC$

$m\angle BAD = m\angle BAC - m\angle DAC$

$x = m\angle BCA - m\angle DAC$

$x = y - m\angle DAC$

Since $m\angle DAC \neq 0$, $y > x$.

Quantity B is greater: **B**

PROBLEM SET 5

1. $T = \frac{kM}{E}$

$5 = \frac{k(1000)}{2}$

$1000k = 10$

$k = \frac{1}{100}$

$T = \frac{\frac{1}{100}(3000)}{3} = \frac{30}{3} = \mathbf{10\ days}$

2. $C = mF + b$

$12 = 10m + b$

$-\ 6 = 4m + b$

$6 = 6m$

$m = 1$

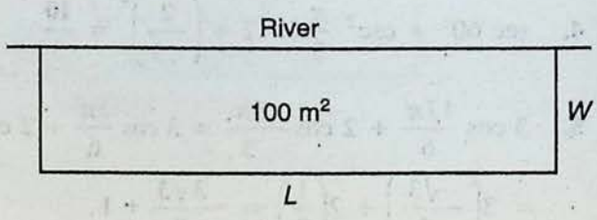
$b = 2$

$C = F + 2$

$C = 9$

$9 million

3.



$LW = 100$

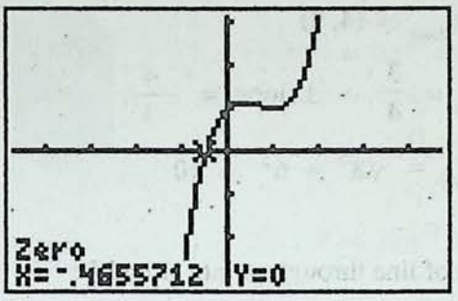
$W = \frac{100}{L}$

$F = L + 2W$

$F = L + 2\left(\frac{100}{L}\right)$

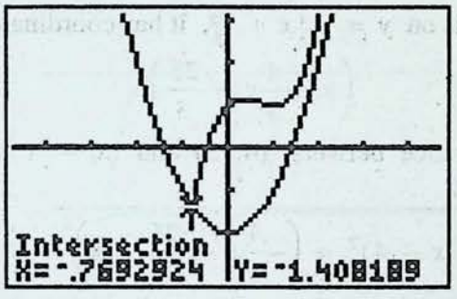
$F = \left(L + \frac{\mathbf{200}}{L}\right)\mathbf{m}$

4.



$x \approx -0.4655712$

5. Let $Y_1 = X^3 - 2X^2 + X + 1$ and $Y_2 = X^2 - 2$.



(−0.7692924, −1.408189)

6. $50° \times \dfrac{\pi}{180°} \approx \mathbf{0.8727}$

7. $2\cos -\dfrac{5\pi}{4} - \sec \dfrac{\pi}{4} = 2\left(-\dfrac{\sqrt{2}}{2}\right) - \dfrac{2}{\sqrt{2}}$

$= -\sqrt{2} - \sqrt{2} = \mathbf{-2\sqrt{2}}$

8. $\tan^2 \dfrac{\pi}{3} - \cot^2 \dfrac{\pi}{3} = (\sqrt{3})^2 - \left(\dfrac{1}{\sqrt{3}}\right)^2 = 3 - \dfrac{1}{3}$

$= \mathbf{\dfrac{8}{3}}$

9. $\sin^2 -\dfrac{2\pi}{3} - \csc -\dfrac{\pi}{2} = \left(\dfrac{-\sqrt{3}}{2}\right)^2 - (-1)$

$= \dfrac{3}{4} + 1 = \mathbf{\dfrac{7}{4}}$

10. $(\sin^2 x)(\csc x)(\cos x) = \sin^2 x \left(\dfrac{1}{\sin x}\right)(\cos x)$

$= \mathbf{\sin x \cos x}$

11. $\dfrac{\cos \alpha \sec \alpha}{\csc \alpha} = \dfrac{\cos \alpha \left(\dfrac{1}{\cos \alpha}\right)}{\dfrac{1}{\sin \alpha}} = \dfrac{1}{\dfrac{1}{\sin \alpha}}$

$= \mathbf{\sin \alpha}$

12. If the polygon does not have four sides, then the polygon is not a triangle.

13. $2x^2 - 3x + 1 = 0$

$$x = \frac{3 \pm \sqrt{9 - 4(2)(1)}}{4}$$

$$x = \mathbf{1, \frac{1}{2}}$$

14. Parallel to the y-axis implies a vertical line. Vertical lines have equations of the form $x = c$, where c is some constant. The vertical line through the point $(2, 3)$ must be $x = 2$, or in general form $\mathbf{x - 2 = 0}$.

15. $\begin{cases} y = x^2 + 1 \\ y = 2x \end{cases}$

$x^2 + 1 = 2x$

$x^2 - 2x + 1 = 0$

$$x = \frac{2 \pm \sqrt{4 - 4(1)(1)}}{2}$$

$x = 1$

$y = 2(1) = 2$

(1, 2)

16. $x^2 = \sqrt{y + 1}$

$x^4 = y + 1$

$\mathbf{y = x^4 - 1}$

17. $\dfrac{x^3 - a^3}{x - a} = \dfrac{(x - a)(x^2 + ax + a^2)}{x - a}$

$= \mathbf{x^2 + ax + a^2}$

18. $\dfrac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} = \dfrac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}\left(\dfrac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}}\right)$

$= \dfrac{3 + 2\sqrt{6} + 2}{3 - 2} = \mathbf{5 + 2\sqrt{6}}$

19. $\dfrac{4}{m + \dfrac{a}{x - 1}} = \dfrac{4}{\dfrac{mx - m + a}{x - 1}} = \mathbf{\dfrac{4x - 4}{mx - m + a}}$

20. $\dfrac{18!}{16!\,2!} = \dfrac{18 \cdot 17 \cdot 16!}{16! \cdot 2} = 9 \cdot 17 = \mathbf{153}$

21. $\displaystyle\sum_{n=1}^{4} [(-2)^n + 1] = -1 + 5 - 7 + 17 = \mathbf{14}$

22. $A = \pi r^2$

$9\pi = \pi r^2$

$r = 3$ cm

$V = \frac{1}{3}\pi r^2 h$

$12\pi = \frac{1}{3}\pi(9)h$

$h = 4$ cm

$l^2 = r^2 + h^2$

$l^2 = 9 + 16$

$l = 5$ cm

$SA = \pi r^2 + \pi rl = 9\pi + 15\pi$

$= \mathbf{24\pi\ cm^2 \approx 75.3982\ cm^2}$

23. (a) $y = -\frac{4}{3}x + \frac{25}{3}$

slope $= -\frac{4}{3}$ $\perp$slope $= \mathbf{\frac{3}{4}}$

(b) $y + 4 = \frac{3}{4}(x - 3)$

$y = \frac{3}{4}x - \frac{9}{4} - 4$

$y = \mathbf{\frac{3}{4}x - \frac{25}{4}}$

(c) $-\frac{4}{3}x + \frac{25}{3} = \frac{3}{4}x - \frac{25}{4}$

$\frac{175}{12} = \frac{25}{12}x$

$x = 7$

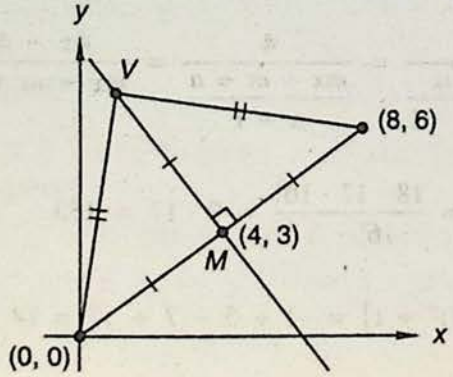
$y = \frac{3}{4}(7) - \frac{25}{4} = \frac{21}{4} - \frac{25}{4} = -1$

(7, –1)

(d) $d = \sqrt{(7-3)^2 + (-1+4)^2} = \sqrt{4^2 + 3^2}$

$= \sqrt{16+9} = \sqrt{25} = \mathbf{5\ units}$

24.



Midpoint$_{Base}$ = (4, 3)

Slope$_{Base}$ $= \frac{3}{4}$ $\perp$slope $= -\frac{4}{3}$

Length$_{Base}$ $= \sqrt{8^2 + 6^2} = 10$

$MV = 5$

Equation of line through points M and V:

$y - 3 = -\frac{4}{3}(x - 4)$

$y = -\frac{4}{3}x + \frac{25}{3}$

Since V is on $y = -\frac{4}{3}x + \frac{25}{3}$, it has coordinates

$$\left(x, -\frac{4}{3}x + \frac{25}{3}\right)$$

The distance between (4, 3) and $(x, -\frac{4}{3}x + \frac{25}{3})$ equals 5.

$$5 = \sqrt{(x-4)^2 + \left(-\frac{4}{3}x + \frac{25}{3} - 3\right)^2}$$

$$25 = (x-4)^2 + \left(-\frac{4}{3}x + \frac{16}{3}\right)^2$$

$$25 = x^2 - 8x + 16 + \frac{16}{9}x^2 - \frac{128}{9}x + \frac{256}{9}$$

$$25 = \frac{25}{9}x^2 - \frac{200}{9}x + \frac{400}{9}$$

$$0 = \frac{25}{9}x^2 - \frac{200}{9}x + \frac{175}{9}$$

$0 = 25x^2 - 200x + 175$

$0 = x^2 - 8x + 7$

$0 = (x - 7)(x - 1)$

$x = 7, 1$

If $x = 7$, then $y = -1$, which is in the fourth quadrant. If $x = 1$, then $y = 7$. The coordinates of the third vertex are **(1, 7)**.

25. If $x < y$, $x < 0$, and $y < 0$, then $-x > -y$.

Quantity A is greater: **A**

Problem Set 6

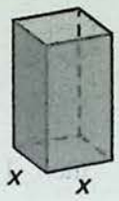
1. $\frac{PV}{T} = \frac{PV}{T}$

$\frac{5(5)}{100} = \frac{4P}{1000}$

$400P = 25{,}000$

$P = \mathbf{62.5\ newtons\ per\ square\ meter}$

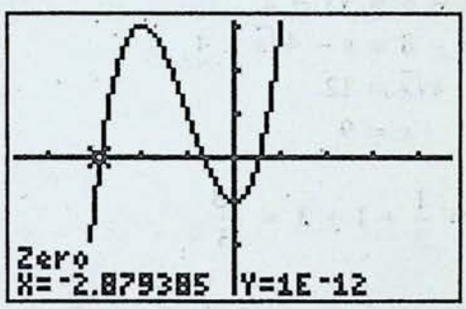
2.



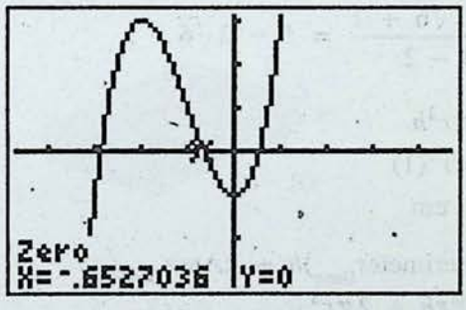
$$A = 2x^2 + 4xh$$
$$100 = 2x^2 + 4xh$$
$$100 - 2x^2 = 4xh$$
$$h = \frac{100 - 2x^2}{4x}$$
$$h = 25x^{-1} - \frac{1}{2}x$$

$$V = x^2h$$
$$V = x^2\left(25x^{-1} - \frac{1}{2}x\right)$$
$$V = 25x - \frac{1}{2}x^3$$

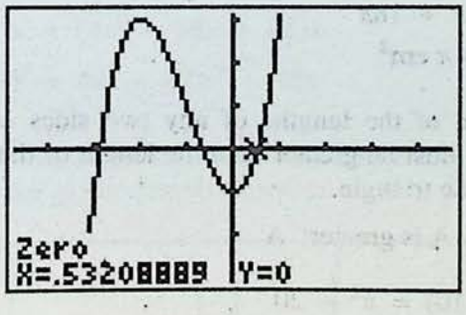
3.



$x \approx -2.879385$

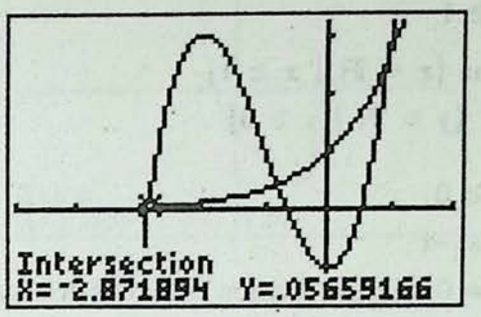


$x \approx -0.6527036$

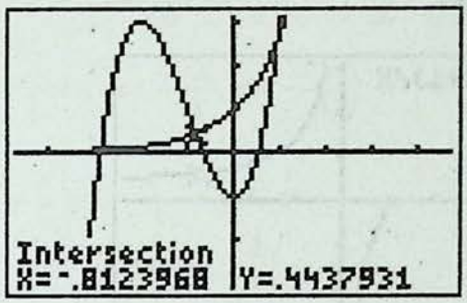


$x \approx 0.53208889$

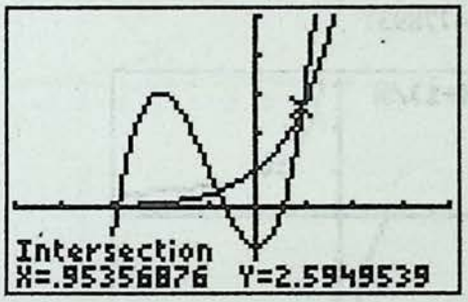
4. Let Y1=X^3+3X^2-1 and Y2=e^(X).



(–2.871894, 0.05659166)



(–0.8123968, 0.4437931)



(0.95356876, 2.5949539)

5. $1.570796327 \times \frac{180°}{\pi} = 90°$

6. Choice **B** is correct because every x-value is mapped to exactly one y-value.

7. (a) $\psi(1) \approx 3$

(b) $\psi(-1) \approx 0$

(c) $\psi(-2) \approx 1$

8. When $f(x) = x^2 + 1$, appropriate y-values are obtained for the given x-values.

The correct choice is **C.**

9.
$$f(x) = 2x^2 - 1$$
$$f(x + \Delta x) = 2(x + \Delta x)^2 - 1$$
$$f(x + \Delta x) = 2[x^2 + 2x(\Delta x) + (\Delta x)^2] - 1$$
$$f(x + \Delta x) = 2x^2 + 4x(\Delta x) + 2(\Delta x)^2 - 1$$

10. $x - 1 \geq 0$

$x \geq 1$

Domain: $\{x \in \mathbb{R} \mid x \geq 1\}$

Range: $\{y \in \mathbb{R} \mid y \geq 0\}$

11. $x + 1 \geq 0$

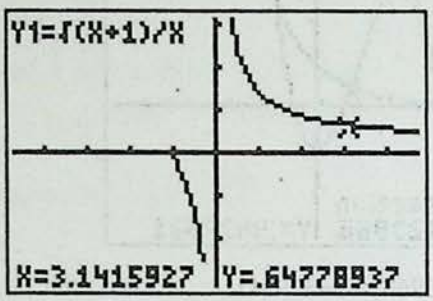
$x \geq -1$

But $x \neq 0$ because division by zero is disallowed.

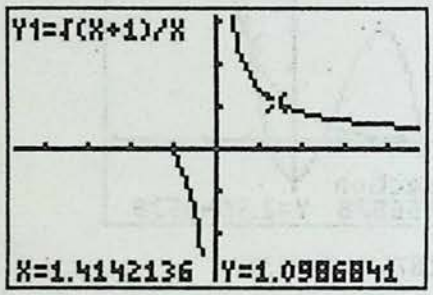
Domain: $\{x \in \mathbb{R} \mid x \geq -1, x \neq 0\}$

Range: $\mathbb{R}$

12.



$y \approx 0.64778937$



$y \approx 1.0986841$

13. $2\cos^2 -\frac{5\pi}{4} - \sec^2 \frac{\pi}{4} = 2\left(-\frac{\sqrt{2}}{2}\right)^2 - \sqrt{2}^2$

$= 2\left(\frac{1}{2}\right) - 2 = -1$

14. $\cot \frac{\pi}{6} + \sin -\frac{\pi}{3} = \frac{3}{\sqrt{3}} + \left(-\frac{\sqrt{3}}{2}\right)$

$= \sqrt{3} - \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

15. $\sin \frac{\pi}{6} \cos -\frac{\pi}{3} = \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$

16. $(\cot^2 x)(\sec^2 x)(\sin x)$

$= \frac{\cos^2 x}{\sin^2 x} \cdot \frac{1}{\cos^2 x} \cdot \sin x = \frac{1}{\sin x} = \csc x$

17. $\frac{(\cot \theta)(\sec \theta)}{(\csc \theta)} = \frac{\frac{\cos \theta}{\sin \theta}\left(\frac{1}{\cos \theta}\right)}{\csc \theta} = \frac{\csc \theta}{\csc \theta} = 1$

18. Converse: If I live in Oklahoma, then I live in Norman.

Inverse: If I do not live in Norman, then I do not live in Oklahoma.

19. The product of the slopes of perpendicular lines is always -1 because perpendicular slopes are opposite reciprocals. Therefore $mn = -1$.

20. $\sqrt{s} - \sqrt{s-8} = 2$

$\sqrt{s-8} = \sqrt{s} - 2$

$s - 8 = s - 4\sqrt{s} + 4$

$4\sqrt{s} = 12$

$s = 9$

21. $\sum_{i=-1}^{1} 3^i = \frac{1}{3} + 1 + 3 = \frac{13}{3}$

22. $\frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}} = \left(\frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}\right)\left(\frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}}\right)$

$= \frac{3 - 2\sqrt{6} + 2}{3 - 2} = 5 - 2\sqrt{6}$

23. $V = \pi r^2 h$

$9\pi = \pi r^2 (1)$

$r = 3$ cm

$SA = (\text{Perimeter}_{\text{Base}})h + 2\text{Area}_{\text{Base}}$

$= 2\pi rh + 2\pi r^2$

$= 2\pi(3)(1) + 2\pi(9)$

$= 6\pi + 18\pi$

$= 24\pi$ cm^2

24. The sum of the lengths of any two sides of any triangle must be greater than the length of the third side of the triangle.

Quantity A is greater: **A**

25. $2(5x - 10) = x^2 - 20$

$10x - 20 = x^2 - 20$

$x^2 - 10x = 0$

$x(x - 10) = 0$

$x = 0, 10$

If $x = 0$ then both the angle and the arc have negative measures, therefore the only acceptable answer is $x = 10$.

Problem Set 7

1. $A = \dfrac{kE}{T}$

$5 = \dfrac{k(20)}{8}$

$k = 2$

$A = \dfrac{2(12)}{6} = 4$

2. $W = mF + b$

$170 = 10m + b$

$170 - 95 = (10m + b) - (5m + b)$

$75 = 5m$

$m = 15$

$b = 20$

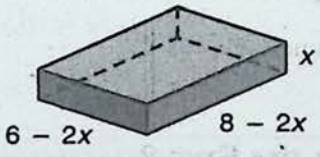
$W = 15F + 20$

$50 = 15F + 20$

$30 = 15F$

$F = 2$

3.



$V = lwh$

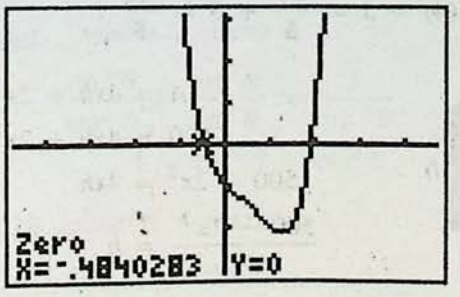
$V = (8 - 2x)(6 - 2x)x$

$V = (48 - 16x - 12x + 4x^2)x$

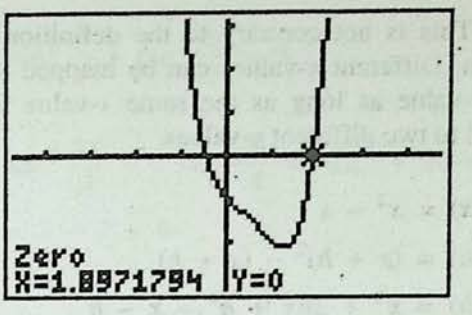
$V = (4x^2 - 28x + 48)x$

$V = 4x^3 - 28x^2 + 48x$

4. This problem is equivalent to finding the zeros of the given quartic equation.



$x \approx -0.4840283$

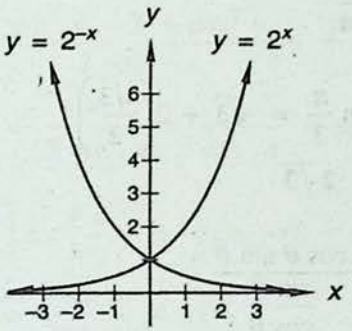


$x \approx 1.8971794$

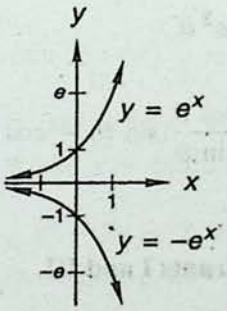
5. $P_1 = \left(\cos\dfrac{\pi}{6}, \sin\dfrac{\pi}{6}\right) = \left(\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$

$P_2 = \left(\cos -\dfrac{2\pi}{3}, \sin -\dfrac{2\pi}{3}\right) = \left(-\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}\right)$

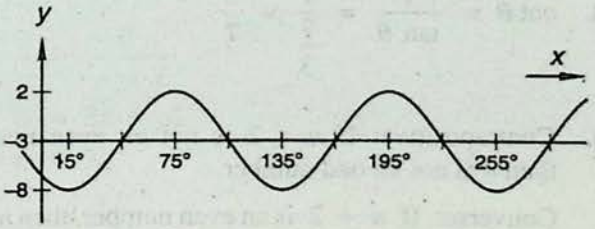
6.



7.



8.



9. Centerline: 2 Phase: $-\dfrac{\pi}{2}$ or $\dfrac{\pi}{2}$

Amplitude: 3 Period: 2π

$y = 2 + 3\cos\left(x + \dfrac{\pi}{2}\right)$ or

$y = 2 - 3\cos\left(x - \dfrac{\pi}{2}\right)$

10. **False.** This is not contrary to the definition of a function. Different x-values can be mapped to the same y-value as long as the same x-value is not mapped to two different y-values.

11. $f(x) = x^2 - x$

$f(x + h) = (x + h)^2 - (x + h)$

$f(x + h) = x^2 + 2hx + h^2 - x - h$

12. **Domain:** $\mathbb{R}$

Range: $\{y \in \mathbb{R} \mid -1 \le y \le 1\}$

13. $\sin^2 -\frac{\pi}{4} \cos^2 \frac{3\pi}{4} = \left(-\frac{\sqrt{2}}{2}\right)^2 \left(-\frac{\sqrt{2}}{2}\right)^2$

$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$

14. $\tan -\frac{2\pi}{3} + 2 \sin \frac{\pi}{3} = \sqrt{3} + 2\left(\frac{\sqrt{3}}{2}\right)$

$= \sqrt{3} + \sqrt{3} = 2\sqrt{3}$

15. $\frac{\cos\theta \sin\theta}{\tan\theta} = \frac{\cos\theta \sin\theta}{\frac{\sin\theta}{\cos\theta}}$

$= \frac{\cos\theta \sin\theta \cos\theta}{\sin\theta} = \cos^2\theta$

16. $(\cot\theta)(\sin\theta) - \cos\theta = \frac{\cos\theta}{\sin\theta}(\sin\theta) - \cos\theta$

$= \cos\theta - \cos\theta = 0$

17. Tangent is positive in **quadrants I and III.**

18. $\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{7}{3}} = \frac{3}{7}$

19. Contrapositive: If $n + 2$ is not an even number, then n is not an odd number.

Converse: If $n + 2$ is an even number, then n is an odd number.

Inverse: If n is not an odd number, then $n + 2$ is not an even number.

20. $x^2 + y^2 = 9$

$1 + y^2 = 9$

$y^2 = 8$

$y = \pm 2\sqrt{2}$

21. $\frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}}{h}$

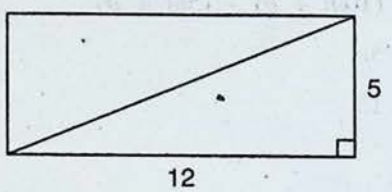
$= \frac{\frac{-h}{x(x+h)}}{h} = -\frac{1}{x(x+h)}$

22. Both e and π are irrational, so they cannot be represented as a ratio of whole numbers.

Choice **B** is correct.

23. **No.** The input value of 1 is mapped to two different output values.

24.



$d^2 = 12^2 + 5^2$

$d = 13$

25. The sum of the measures of any two angles of any triangle is equal to the exterior angle of the triangle's third angle.

The quantities are equal: **C**

Problem Set 8

1. $D = mx + b$

$5 = 0m + b$

$b = 5$

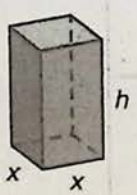
$17 = 10m + 5$

$12 = 10m$

$m = \frac{6}{5}$

$D = \frac{6}{5}(4) + 5 = \frac{24}{5} + 5 = \frac{49}{5}$

2.



$A = 4xh + 2x^2$

$500 = 4xh + 2x^2$

$500 - 2x^2 = 4xh$

$\frac{500 - 2x^2}{4x} = h$

$125x^{-1} - \frac{1}{2}x = h$

$V = x^2h$

$V = x^2\left(125x^{-1} - \frac{1}{2}x\right)$

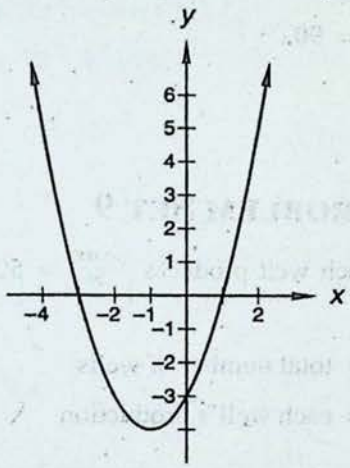
$V = 125x - \frac{1}{2}x^3$

3. (a) $y = x^2 + 2x - 3$

$y = (x^2 + 2x + 1) - 3 - 1$

$y = (x + 1)^2 - 4$

(b)

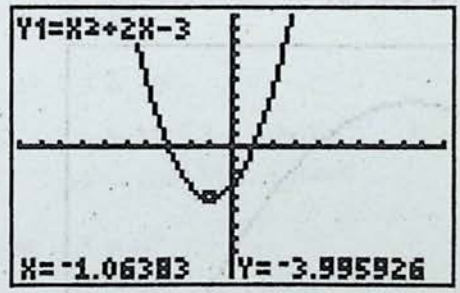


(c) The parabola opens **upward.**

(d) $x = -1$

(e) $(-1, -4)$

4.



$(-1.0638, -3.9959)$ in ZStandard

Answers may vary.

5. (a) $\sin^2\theta + \cos^2\theta = 1$

$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$1 + \cot^2\theta = \csc^2\theta$

(b) $\sin^2\theta + \cos^2\theta = 1$

$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$\tan^2\theta + 1 = \sec^2\theta$

6. $\sin^2\frac{\pi}{7} + \cos^2\frac{\pi}{7} = 1$

7. $\sec^2\frac{5\pi}{4} + 2\tan -\frac{\pi}{4} = \sqrt{2}^2 + 2(-1)$

$= 2 - 2 = 0$

8. $\sin -\theta = -\sin\theta = \frac{4}{5}$

9. $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta = -\frac{4}{5}$

10. $\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta = \frac{1}{\sin\theta} = -\frac{5}{4}$

11. $\frac{\sin^2 x + 2 + \cos^2 x}{3\csc^2 -x} = \frac{\sin^2 x + \cos^2 x + 2}{3(-\csc x)^2}$

$= \frac{1 + 2}{3\csc^2 x} = \frac{1}{\csc^2 x} = \sin^2 x$

12. $\left[\sec\left(\frac{\pi}{2} - x\right)\right](\sin -x) = \csc x\,(-\sin x)$

$= \frac{-\sin x}{\sin x} = -1$

13. $(\sin x)\left[\cos\left(\frac{\pi}{2} - x\right)\right] + (\cos -x)(\cos x)$

$= (\sin x)(\sin x) + (\cos x)(\cos x)$

$= \sin^2 x + \cos^2 x = 1$

14. $x^2 - 3x + 2 = 0$

$(x - 2)(x - 1) = 0$

$x = 2, 1$

15. $\frac{3}{x} = \frac{7}{x + L}$

$7x = 3x + 3L$

$4x = 3L$

$x = \frac{3}{4}L$

16. $\frac{a}{h} = \frac{4}{x + h}$

$4h = ax + ah$

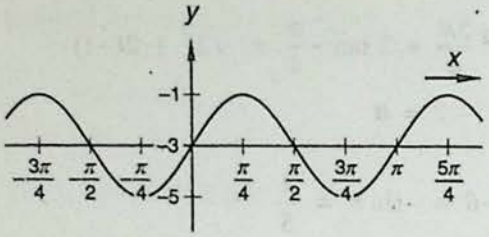
$4h - ah = ax$

$h(4 - a) = ax$

$h = \frac{ax}{4 - a}$

17. $\left(\cos -\frac{\pi}{3}, \sin -\frac{\pi}{3}\right) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

18.



19. Centerline: -1 Period: 2π

Amplitude: 11 Phase: $\frac{\pi}{2}$ or $-\frac{\pi}{2}$

$y = 1 + 11 \sin\left(\theta - \frac{\pi}{2}\right)$ or

$y = 1 - 11 \sin\left(\theta + \frac{\pi}{2}\right)$

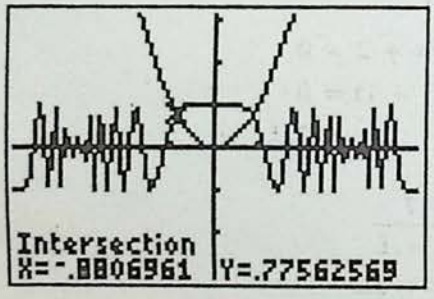
20. In both B and D each x-value is mapped to exactly one y-value.

Choices **B** and **D** are correct.

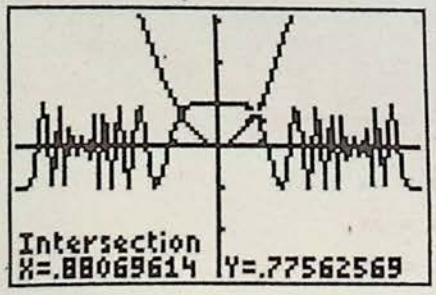
21. $\frac{(x+h)^2 - x^2}{h} = \frac{x^2 + 2hx + h^2 - x^2}{h}$

$= \frac{2hx + h^2}{h} = \frac{h(2x+h)}{h} = \mathbf{2x + h}$

22. Let $Y_1 = \cos(X^3)$ and $Y_2 = X^2$.



(–0.8806961, 0.77562569)



(0.88069614, 0.77562569)

23. $f(x) = 2(x + 3)(x + 2)$

$f(x) = 2(x^2 + 5x + 6)$

$\boldsymbol{f(x) = 2x^2 + 10x + 12}$

24. If $y > 0$, then $x > z$. If $y < 0$, then $x < z$.

Insufficient information: **D**

25. If $a + b = 10$, then $a^2 + 2ab + b^2 = 100$.

Substituting $ab = 5$ gives $a^2 + 10 + b^2 = 100$

or $\boldsymbol{a^2 + b^2 = 90}$.

Problem Set 9

1. Originally, each well produces $\frac{10{,}000}{20} = 500$ barrels per day.

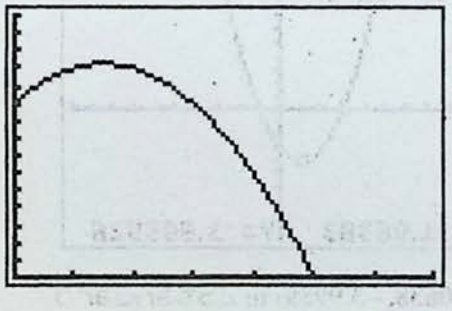
$20 + x$ = total number of wells

$500 - 10x$ = each well's production

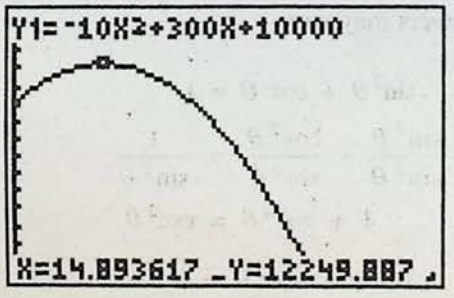
$V = (20 + x)(500 - 10x)$

$\boldsymbol{V = 10{,}000 + 300x - 10x^2}$

2. (a) Set Xmin=0, Xmax=70, Xscl=10, Ymin=0, Ymax=15000, Yscl=1000, and Xres=1.

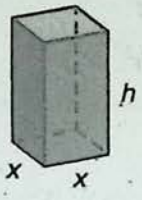


(b)



(c) **15 additional wells**

(d) **12,250 barrels of oil per day**

3. 

$$V = x^2h$$
$$125 = x^2h$$
$$h = \frac{125}{x^2}$$
$$h = 125x^{-2}$$

$$A = x^2 + x^2 + 4xh$$
$$C = 5x^2 + 2x^2 + 2(4xh)$$
$$C = 7x^2 + 8xh$$
$$C = 7x^2 + 8x(125x^{-2})$$
$$\boldsymbol{C = 7x^2 + 1000x^{-1}}$$

4. $y = x^2 - 3x + 4$

$$y = \left(x^2 - 3x + \frac{9}{4}\right) + 4 - \frac{9}{4}$$
$$y = \left(x - \frac{3}{2}\right)^2 + \frac{7}{4}$$

Vertex: $\left(\frac{3}{2}, \frac{7}{4}\right)$

Axis of symmetry: $x = \frac{3}{2}$

5. $x_m = \frac{-5 + 0}{2} = -2.5$ $\qquad y_m = \frac{-8 + 4}{2} = -2$

(–2.5, –2)

6. (a) $7.3 = 10^L$

$L = \log 7.3 \approx 0.8633$

$7.3 = 10^{\log 7.3} \approx \mathbf{10^{0.8633}}$

(b) $7.3 = e^L$

$L = \ln 7.3 \approx 1.9879$

$7.3 = e^{\ln 7.3} \approx \mathbf{e^{1.9879}}$

7. If $3^y = 4$, then $\log_3 4 = y$.

Choice **B** is correct.

8. $\frac{y^3y^{3/4-2}z^2}{y^{(3-2)3}z^{(3-2)6}} = y^{3+3/4-2-1/3}z^{2-1/6}$

$= \mathbf{y^{17/12}z^{11/6}}$

9. (a) If $10^x = 3$, then $x = \mathbf{\log 3 \approx 0.4771}$.

(b) If $e^x = 5$, then $x = \mathbf{\ln 5 \approx 1.6094}$.

10. $\log_3 27 = 2b + 1$

$3^{2b+1} = 27$

$3^{2b+1} = 3^3$

$2b + 1 = 3$

$2b = 2$

$b = \mathbf{1}$

11. $\log_x (3x - 2) = 2$

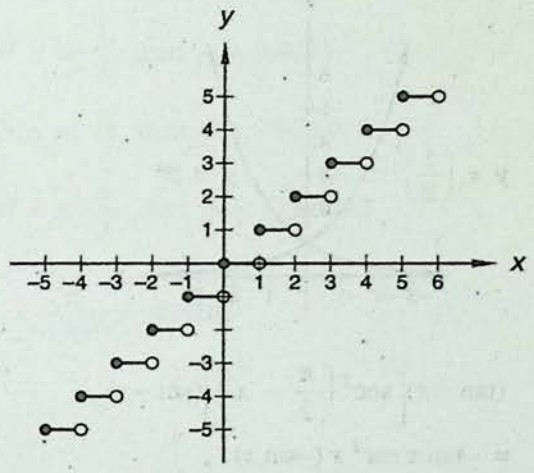
$x^2 = 3x - 2$

$x^2 - 3x + 2 = 0$

$(x - 2)(x - 1) = 0$

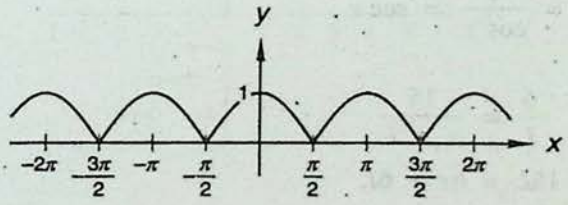
$x = \mathbf{2, 1}$

12. (a)

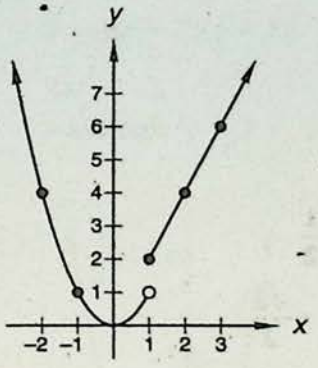


(b) $\boldsymbol{f(1.2) = 1; f(-1.2) = -2}$

13.



14.



15. "x is less than 0.001 away from 3."

$\boldsymbol{\{x \in \mathbb{R} \mid 2.999 < x < 3.001\}}$

16. $f(x) = \begin{cases} -2 & \text{when } x < 0 \\ x - 1 & \text{when } 0 \le x \le 3 \\ 1 & \text{when } x > 3 \end{cases}$

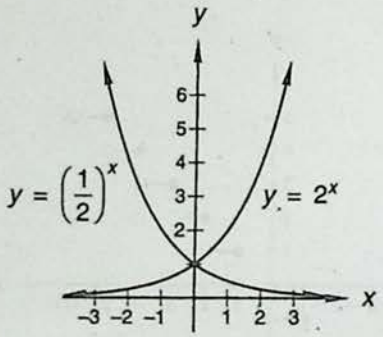
17. Centerline: -5 Period: 3π . $C = \frac{2}{3}$

Amplitude: 4 Phase: $-\frac{5\pi}{4}, \frac{\pi}{4}$

$y = -5 + 4\sin\left[\frac{2}{3}\left(x + \frac{5\pi}{4}\right)\right]$ or

$y = -5 - 4\sin\left[\frac{2}{3}\left(x - \frac{\pi}{4}\right)\right]$

18.



19. $(\tan -x)\left[\sec^2\left(\frac{\pi}{2} - x\right)\right](\sin -x)$

$= -\tan x \csc^2 x (-\sin x)$

$= \frac{\sin x}{\cos x} \cdot \frac{1}{\sin^2 x} \cdot \sin x$

$= \frac{1}{\cos x} = \sec x$

20. $\frac{6}{L} = \frac{15}{x + L}$

$15L = 6x + 6L$

$9L = 6x$

$L = \frac{2}{3}x$

21. $y_1 = \sin\frac{\pi}{6} = \frac{1}{2}$

$y_2 = \sin 210° = -\frac{1}{2}$

$y_3 = \sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

22. The mapping of f **is not a function.** Each value of x is mapped to two different values: the positive and negative square root of x.

23. $f(x + h) - f(x) = (x + h)^2 - x^2$

$= x^2 + 2hx + h^2 - x^2$

$= 2hx + h^2$

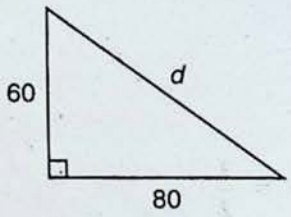
24. $\frac{\sum_{i=1}^{10} i}{10} = \frac{55}{10} = 5.5$

25. This is a geometric representation of the Pythagorean Theorem; the sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse of the triangle.

The quantities are equal: **C**

Problem Set 10

1.

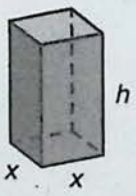


$d^2 = 60^2 + 80^2$

$d^2 = 10{,}000$

$d = \mathbf{100\ miles}$

2. (a)



$V = x^2h$

$100 = x^2h$

$h = \frac{100}{x^2}$

$h = 100x^{-2}$

$A = 2x^2 + 4xh$

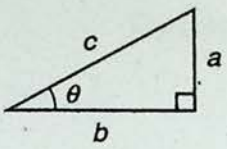
$A = 2x^2 + 4x(100x^{-2})$

$A = \mathbf{2x^2 + 400x^{-1}}$

(b) x must be greater than zero because it is the length of a side.

$\{x \in \mathbb{R} \mid x > 0\}$

3.



$$\sin^2\theta + \cos^2\theta = \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2$$

$$= \frac{a^2 + b^2}{c^2} = \frac{c^2}{c^2} = 1$$

Therefore, $\mathbf{\sin^2\theta + \cos^2\theta = 1.}$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

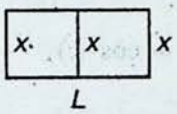
$$\mathbf{1 + \cot^2\theta = \csc^2\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\mathbf{\tan^2\theta + 1 = \sec^2\theta}$$

4.



$$F = 3x + 2L$$

$$100 = 3x + 2L$$

$$100 - 3x = 2L$$

$$L = 50 - \frac{3}{2}x$$

$$A = xL$$

$$A = x\left(50 - \frac{3}{2}x\right)$$

$$\mathbf{A = 50x - \frac{3}{2}x^2}$$

5. According to the Remainder Theorem $f(1)$ is the value of the remainder when the polynomial is divided by $x - 1$.

$f(x) = x^5 - 2x^4 + x^3 - x^2 + 3x + 1$

$f(1) = 1 - 2 + 1 - 1 + 3 + 1 = 3$

The remainder is **3.**

6. (a) -1

	1	0	−2	2	1
	↓	−1	1	1	−3
	1	−1	−1	3	**−2**

$f(-1) = \mathbf{-2}$

(b) 1

	1	0	−2	2	1
	↓	1	1	−1	1
	1	1	−1	1	**2**

$f(1) = \mathbf{2}$

(c) 3

	1	0	−2	2	1
	↓	3	9	21	69
	1	3	7	23	**70**

$f(3) = \mathbf{70}$

7. Possible rational zeros:

$$\frac{\pm1, \pm2, \pm4}{\pm1} = \pm1, \pm2, \pm4$$

8. 1

	1	−1	−4	4
	↓	1	0	−4
	1	0	−4	**0**

$f(x) = (x - 1)(x^2 - 4)$

$= (x - 1)(x + 2)(x - 2)$

Roots: **1, 2, −2**

9. Set `Y1=X^4-22X^3+πX^2-X+√(2)` and then use the `1:value` feature in the CALCULATE menu.

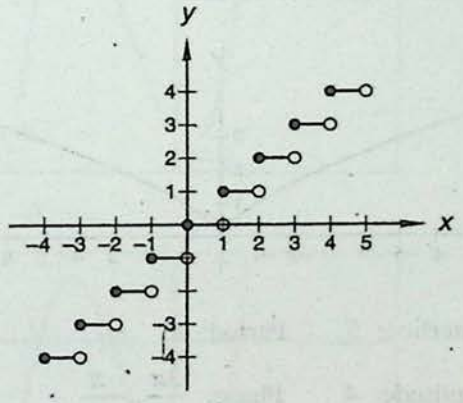
(a) If $x = \frac{1}{3}$, then $y \approx \mathbf{0.6275}$

(b) If $x = \sqrt{3}$, then $y \approx \mathbf{-96.2084}$

(c) If $x = \frac{\pi}{2}$, then $y \approx \mathbf{-71.5842}$

10. $47° \times \frac{\pi}{180°} \approx \mathbf{0.8203}$

11. (a)

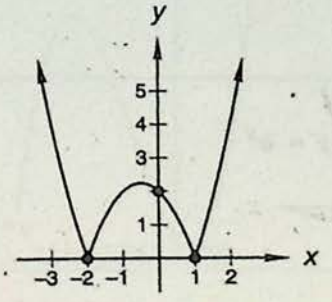


(b) $y = |x^2 + x - 2|$

$y = |(x + 2)(x - 1)|$

Zeros: $-2, 1$

y-intercept: $y = 2$



12. $2x + 1 = \log_{1/3} 9$

$$\left(\frac{1}{3}\right)^{2x+1} = 9$$
$$(3^{-1})^{2x+1} = 3^2$$
$$3^{-2x-1} = 3^2$$
$$-2x - 1 = 2$$
$$-2x = 3$$
$$x = -\frac{3}{2}$$

13. $\ln b^3 = 2$

$$b^3 = e^2$$
$$(b^3)^{1/3} = (e^2)^{1/3}$$
$$b = e^{2/3} \approx \mathbf{1.9477}$$

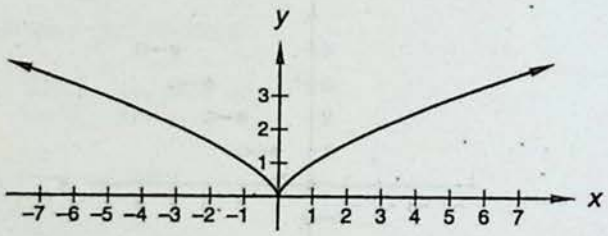
14. (a) $10^x = 4$

$$\log 10^x = \log 4$$
$$x = \mathbf{\log 4 \approx 0.6021}$$

(b) $e^x = 4$

$$\ln e^x = \ln 4$$
$$x = \mathbf{\ln 4 \approx 1.3863}$$

15.

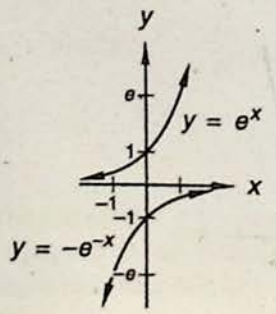


16. Centerline: 5 Period: π

Amplitude: 4 Phase: $\frac{3\pi}{8}, -\frac{\pi}{8}$

$$y = 5 + 4\sin\left[2\left(x - \frac{3\pi}{8}\right)\right] \text{ or}$$
$$y = 5 - 4\sin\left[2\left(x + \frac{\pi}{8}\right)\right]$$

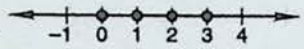
17.



18. $|2x - 3| < 4$

$$\left|2\left(x - \frac{3}{2}\right)\right| < 4$$
$$2\left|x - \frac{3}{2}\right| < 4$$
$$\left|x - \frac{3}{2}\right| < 2$$

"x is less than 2 away from $\frac{3}{2}$."



19. $\sec\alpha = \dfrac{\sqrt{a^2 + b^2}}{b}$

20. $\dfrac{\sin^2 -\theta + \cos^2 -\theta + 2}{3\tan -\theta} = \dfrac{1 + 2}{3\tan -\theta}$

$$= \frac{1}{\tan -\theta} = \frac{1}{-\tan\theta} = -\cot\theta$$

21. $\sin x - \sin x\cos^2 x = \sin x(1 - \cos^2 x)$

$$= \sin x(\sin^2 x) = \sin^3 x$$

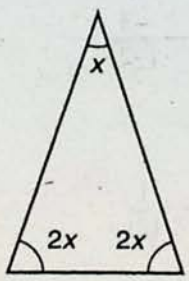
22. $f(x) = c(x - 2)(x + 3)$

$$6 = c(3 - 2)(3 + 3)$$
$$6 = 6c$$
$$c = 1$$
$$f(x) = (x - 2)(x + 3)$$
$$\mathbf{f(x) = x^2 + x - 6}$$

23. $\dfrac{f(x + \Delta x) - f(x)}{\Delta x} = \dfrac{(x + \Delta x)^2 - x^2}{\Delta x}$

$$= \frac{x^2 + 2x(\Delta x) + (\Delta x)^2 - x^2}{\Delta x}$$
$$= \frac{2x(\Delta x) + (\Delta x)^2}{\Delta x} = \frac{\Delta x(2x + \Delta x)}{\Delta x}$$
$$= \mathbf{2x + \Delta x}$$

24.



$$2x + 2x + x = 180°$$
$$5x = 180°$$
$$x = \mathbf{36°}$$

25. $4x + 60 = (5x - 40) + 3x$

$$4x + 60 = 8x - 40$$
$$100 = 4x$$
$$x = \mathbf{25}$$

Problem Set 11

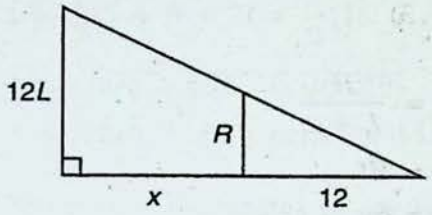
1. $S = \dfrac{k}{X^2}$

$8 = \dfrac{k}{5^2}$

$k = 200$

$S = \dfrac{200}{X^2} = \dfrac{200}{2^2} = \mathbf{50}$

2.



$\dfrac{R}{12} = \dfrac{12L}{x + 12}$

$144L = Rx + 12R$

$Rx = 144L - 12R$

$x = \dfrac{144L - 12R}{R}$

$x = 144LR^{-1} - 12$

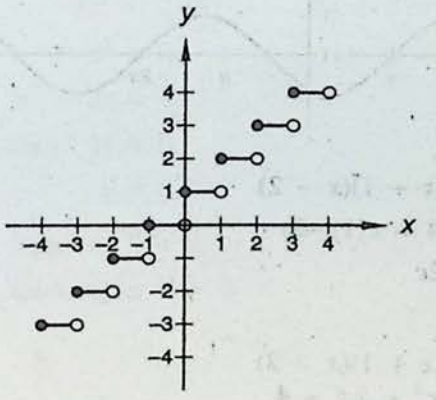
Shadow $= x + 12 = (144LR^{-1} - 12) + 12$

$= \mathbf{144LR^{-1}}$ **in.**

3. (a) $\lim\limits_{x \to 0^+} f(x) = \mathbf{2}$ (b) $\lim\limits_{x \to 0^-} f(x) = \mathbf{1}$

(c) $\lim\limits_{x \to -1^-} f(x) = \mathbf{-1}$ (d) $\lim\limits_{x \to -1^+} f(x) = \mathbf{0}$

4.



(a) $\lim\limits_{x \to 1^+} g(x) = \mathbf{2}$ (b) $\lim\limits_{x \to 1^-} g(x) = \mathbf{1}$

5. (a) $\underline{-1}$

2	−4	3	−2	1	−1
↓	−2	6	−9	11	−12
2	−6	9	−11	12	$\boxed{-13}$

$f(-1) = \mathbf{-13}$

(b) $\underline{2}$

2	−4	3	−2	1	−1
↓	4	0	6	8	18
2	0	3	4	9	$\boxed{17}$

$f(2) = \mathbf{17}$

(c) $\underline{-2}$

2	−4	3	−2	1	−1
↓	−4	16	−38	80	−162
2	−8	19	−40	81	$\boxed{-163}$

$f(-2) = \mathbf{-163}$

6. (a) Possible rational roots: $\dfrac{\pm 1, \pm 3}{\pm 1, \pm 2, \pm 3, \pm 6}$

$= \mathbf{\pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm 3, \pm\frac{3}{2}}$

(b) $\underline{3}$

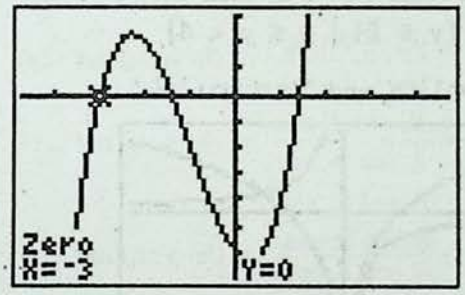
6	−19	2	3
↓	18	−3	−3
6	−1	−1	$\boxed{0}$

$h(x) = (x - 3)(6x^2 - x - 1)$

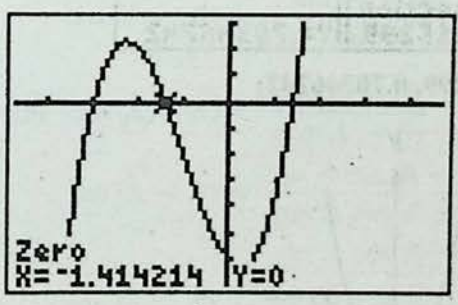
$= (x - 3)(3x + 1)(2x - 1)$

Rational zeros: $\mathbf{3, -\frac{1}{3}, \frac{1}{2}}$

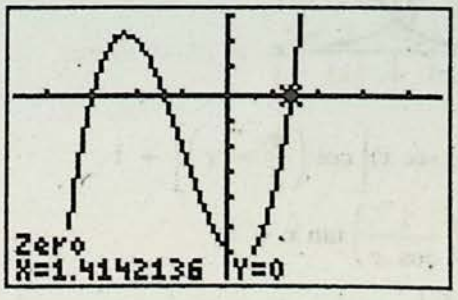
7.



$x = \mathbf{-3}$



$x \approx \mathbf{-1.414214}$



$x \approx \mathbf{1.4142136}$

8. $1 = \log_x(2x - 7)$

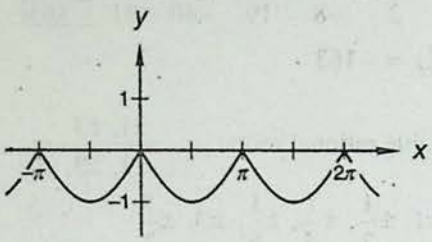
$x^1 = 2x - 7$

$x = 7$

9. $e^x = 10$

$x = \ln 10 \approx 2.3025$

10.



11. $r^2 = (-2 - 1)^2 + (6 - 2)^2$

$r^2 = (-3)^2 + 4^2$

$r^2 = 9 + 16$

$r^2 = 25$

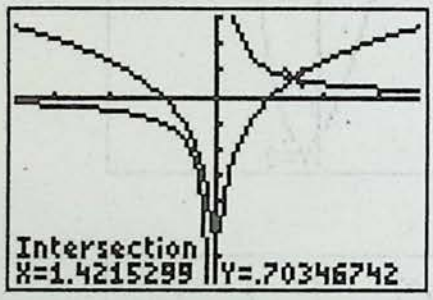
$r = 5$

$5^2 = (x - 1)^2 + (y - 2)^2$

12. If $y = x^2$ and $|x| < 2$, then $0 \le y < 4$.

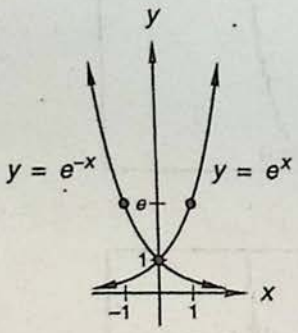
Range: $\{y \in \mathbb{R} \mid 0 \le y < 4\}$

13. Let $Y_1 = 1/X$ and $Y_2 = \ln(X^2)$.



(1.4215299, 0.70346742)

14.



15. $-(\sin -x)(\sec x)\left[\cot\left(\frac{\pi}{2} - x\right)\right] + 1$

$= \sin x \left(\frac{1}{\cos x}\right) \tan x + 1$

$= \left(\frac{\sin x}{\cos x}\right) \tan x + 1 = \tan^2 x + 1 = \sec^2 x$

16. $\frac{\sin x - \sin x \cos^2 x}{\sec^2 x - 1} = \frac{\sin x (1 - \cos^2 x)}{\tan^2 x}$

$= \frac{\sin x \sin^2 x}{\tan^2 x} = \frac{\sin^3 x \cos^2 x}{\sin^2 x}$

$= \sin x \cos^2 x$

17. $y = x^2 - 2x + 4$

$y = (x^2 - 2x + 1) + 4 - 1$

$y = (x - 1)^2 + 3$

Vertex: **(1, 3)**

18. $\frac{6}{L} = \frac{H}{L + x}$

$6L + 6x = HL$

$HL - 6L = 6x$

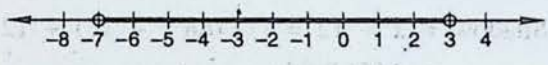
$L(H - 6) = 6x$

$$L = \frac{6x}{H - 6}$$

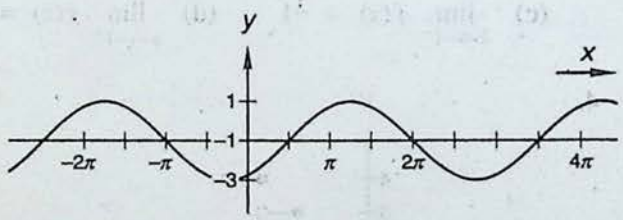
19. $|3x + 6| < 15$

$|x + 2| < 5$

"x is less than 5 away from -2."



20. Period $= \frac{2\pi}{\frac{2}{3}} = 3\pi$



21. $f(x) = c(x + 1)(x - 2)$

$f(0) = -4 = c(1)(-2)$

$-4 = -2c$

$c = 2$

$f(x) = 2(x + 1)(x - 2)$

$f(x) = 2x^2 - 2x - 4$

22. $x^2 - 1 \ge 0$

$x^2 \ge 1$

$|x| \ge 1$

Domain: $\{x \in \mathbb{R} \mid |x| \ge 1\}$

Range: $\{y \in \mathbb{R} \mid y \ge 0\}$

23. $$\frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{\frac{1}{x+\Delta x}-\frac{1}{x}}{\Delta x}$$

$$= \frac{\frac{x-(x+\Delta x)}{x(x+\Delta x)}}{\Delta x} = \frac{\frac{-\Delta x}{x(x+\Delta x)}}{\Delta x}$$

$$= \frac{-1}{x(x+\Delta x)}$$

24. $s = \frac{1}{2}(5+6+7) = \frac{1}{2}(18) = 9$

$A = \sqrt{9(9-5)(9-6)(9-7)}$

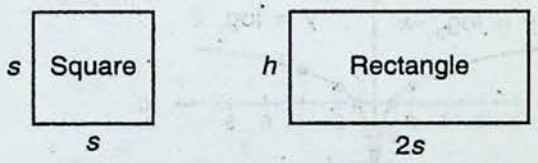
$= \sqrt{216} = \mathbf{6\sqrt{6}\ units^2} \approx \mathbf{14.6969\ units^2}$

25. The sum of the lengths of any two sides of any triangle must be greater than the length of the third side of the triangle.

Quantity B is greater: **B**

Problem Set 12

1. Let s = side of square and h = height of rectangle.



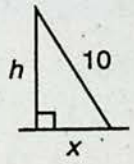
Area: $8s^2 = 2sh$, $8s^2 = 2s(8)$, $s^2 = 2s$, $s^2 - 2s = 0$, $s(s-2) = 0$, $s = 0, 2$

Perimeter: $4s + 16 = 2(2s + h)$, $2s + 8 = 2s + h$, $h = 8$

Square: 2 × 2

Rectangle: 4 × 8

2.



$10^2 = h^2 + x^2$

$h^2 = 100 - x^2$

$\mathbf{h = \sqrt{100 - x^2}\ feet}$

3. For the key trigonometric identities, see section 12.A in the textbook.

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A+A) = \sin A\cos A + \cos A\sin A$

$\mathbf{\sin(2A) = 2\sin A\cos A}$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A+A) = \cos A\cos A - \sin A\sin A$

$\mathbf{\cos(2A) = \cos^2 A - \sin^2 A}$

$\cos(2A) = \cos^2 A - (1 - \cos^2 A)$

$\mathbf{\cos(2A) = 2\cos^2 A - 1}$

$\cos(2A) = (1 - \sin^2 A) - \sin^2 A$

$\mathbf{\cos(2A) = 1 - 2\sin^2 A}$

4. $\cos\alpha = \frac{1}{5}$

$\cos(2\alpha) = 2\cos^2\alpha - 1 = 2\left(\frac{1}{5}\right)^2 - 1$

$= \frac{2}{25} - 1 = -\frac{23}{25}$

5. (a) $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$

$\tan(A+B) = \frac{\sin A\cos B + \cos A\sin B}{\cos A\cos B - \sin A\sin B}$

$\tan(A+B) = \frac{\frac{\sin A\cos B}{\cos A\cos B} + \frac{\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} - \frac{\sin A\sin B}{\cos A\cos B}}$

$\mathbf{\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}}$

(b) $\tan(A-B) = \frac{\sin(A-B)}{\cos(A-B)}$

$\tan(A-B) = \frac{\sin A\cos B - \cos A\sin B}{\cos A\cos B + \sin A\sin B}$

$\tan(A-B) = \frac{\frac{\sin A\cos B}{\cos A\cos B} - \frac{\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} + \frac{\sin A\sin B}{\cos A\cos B}}$

$\mathbf{\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}}$

6. $\tan 75° = \tan(45° + 30°)$

$$= \frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°}$$

$$= \frac{1 + \frac{\sqrt{3}}{3}}{1 - 1\left(\frac{\sqrt{3}}{3}\right)} = \frac{\frac{3 + \sqrt{3}}{3}}{\frac{3 - \sqrt{3}}{3}}$$

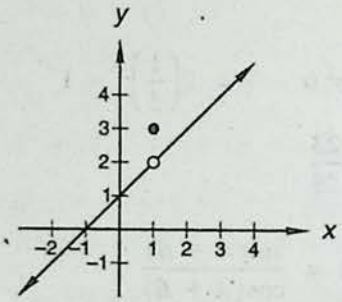
$$= \frac{3 + \sqrt{3}}{3 - \sqrt{3}} = \frac{3 + \sqrt{3}}{3 - \sqrt{3}}\left(\frac{3 + \sqrt{3}}{3 + \sqrt{3}}\right)$$

$$= \frac{9 + 6\sqrt{3} + 3}{9 - 3} = \frac{12 + 6\sqrt{3}}{6} = \mathbf{2 + \sqrt{3}}$$

7. $(\sin x + \cos x)^2 = \sin^2 x + 2\sin x \cos x + \cos^2 x$

$= (\sin^2 x + \cos^2 x) + 2\sin x \cos x = 1 + \sin(2x)$

8.



9. (a) $\lim_{x \to 1^+} f(x) = 2$

(b) $\lim_{x \to 1^-} f(x) = 2$

10. Possible rational roots:

$$\frac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2} = \pm 1, \pm\frac{1}{2}, \pm 2, \pm 4$$

−1	2	−7	−5	4
	↓	−2	9	−4
	2	−9	4	0

$y = 2x^3 - 7x^2 - 5x + 4$

$= (x + 1)(2x^2 - 9x + 4)$

$= (x + 1)(2x - 1)(x - 4)$

Roots: $\mathbf{-1, \frac{1}{2}, 4}$

11. $\log_4(3x + 1) = \frac{1}{2}$

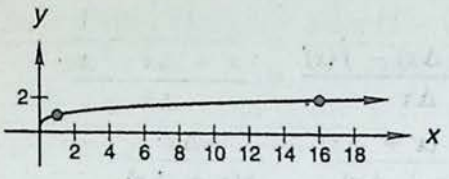
$3x + 1 = 4^{1/2}$

$3x + 1 = 2$

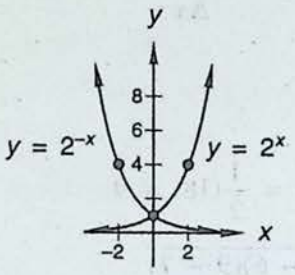
$3x = 1$

$x = \mathbf{\frac{1}{3}}$

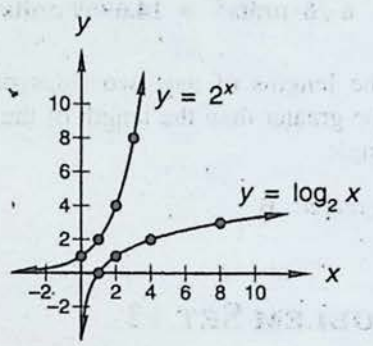
12.



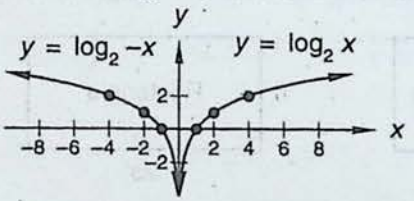
13.



14.



15.



16. $\left[\sin\left(\frac{\pi}{2} - x\right)\right](\csc -x)(\sin x)(\cos -x)$

$= \cos x(-\csc x)\sin x \cos x = \cos x(-1)\cos x$

$= \mathbf{-\cos^2 x}$ or $\mathbf{\sin^2 x - 1}$

17. $f(x) = c(x + 1)(x - 2)$

$f(0) = -2 = c(1)(-2)$

$-2 = -2c$

$c = 1$

$f(x) = (x + 1)(x - 2)$

$\mathbf{f(x) = x^2 - x - 2}$

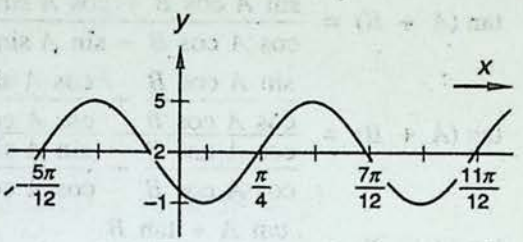
18. $\frac{y}{10 - x} = \frac{5}{10}$

$10y = 5(10 - x)$

$2y = 10 - x$

$\mathbf{y = 5 - \frac{1}{2}x}$

19. Period $= \frac{2\pi}{3}$



20. $\frac{f(x+h)-f(x)}{h} = \frac{2(x+h)^2 - 2x^2}{h}$

$= \frac{2(x^2+2hx+h^2)-2x^2}{h}$

$= \frac{2x^2+4hx+2h^2-2x^2}{h} = \frac{h(4x+2h)}{h}$

$= \mathbf{4x + 2h}$

21. (a) $\cos(2A) = 2\cos^2 A - 1$

$2\cos^2 A = 1 + \cos(2A)$

$\cos^2 A = \frac{1}{2} + \frac{1}{2}\cos(2A)$

Let $A = \frac{x}{2}$.

$\cos^2 \frac{x}{2} = \frac{1}{2} + \frac{1}{2}\cos x$

$\cos \frac{x}{2} = \pm\sqrt{\frac{1}{2} + \frac{1}{2}\cos x}$

(b) $\cos(2A) = 1 - 2\sin^2 A$

$2\sin^2 A = 1 - \cos(2A)$

$\sin^2 A = \frac{1}{2} - \frac{1}{2}\cos(2A)$

Let $A = \frac{x}{2}$.

$\sin^2 \frac{x}{2} = \frac{1}{2} - \frac{1}{2}\cos x$

$\sin \frac{x}{2} = \pm\sqrt{\frac{1}{2} - \frac{1}{2}\cos x}$

22. $1 - x \geq 0$

$1 \geq x$

$\{x \in \mathbb{R} \mid x \leq 1\}$

23. If the sides opposite two angles of a triangle do not have equal lengths, then the two angles do not have equal measures.

24. $\frac{1}{2}x = 75$

$x = 150°$

$\frac{1}{2}z = 35$

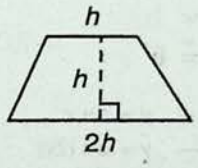
$z = 70°$

$x + y + z = 360°$

$y = 360° - 150° - 70°$

$y = 140°$

25.



$A = \left(\frac{B_1 + B_2}{2}\right)h$

$12 = \left(\frac{h+2h}{2}\right)h$

$12 = \frac{3}{2}h^2$

$h^2 = 8$

$h = 2\sqrt{2}$ units $\approx$ 2.8284 units

Problem Set 13

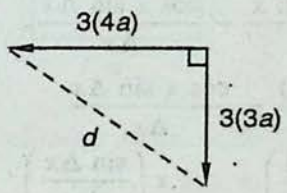
1. $\frac{S}{D^2 W} = \frac{S}{D^2 W}$

$\frac{40}{M^2 P} = \frac{S}{3^2 A}$

$S(M^2 P) = 40(9A)$

$S = \frac{360A}{M^2 P}$

2.



$d^2 = [3(4a)]^2 + [3(3a)]^2$

$d^2 = 144a^2 + 81a^2$

$d^2 = 225a^2$

$d = 15a$ miles

3. $\sin^{-1} -\frac{\sqrt{2}}{2} = -\frac{\pi}{4}$

4. $\cos^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{6}$

5. $\csc x = -2$

$$\sin x = -\frac{1}{2}$$

$$x = 210°, 330°$$

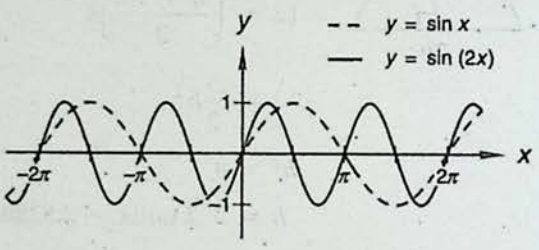
6. $\cos^2 x = 1$

$$\cos x = \pm 1$$

$$x = 0, \pi$$

7.
$$\sin^2 x + 2\cos x - 2 = 0$$
$$(1 - \cos^2 x) + 2\cos x - 2 = 0$$
$$\cos^2 x - 2\cos x + 1 = 0$$
$$(\cos x - 1)(\cos x - 1) = 0$$
$$\cos x = 1$$
$$x = 0$$

8.



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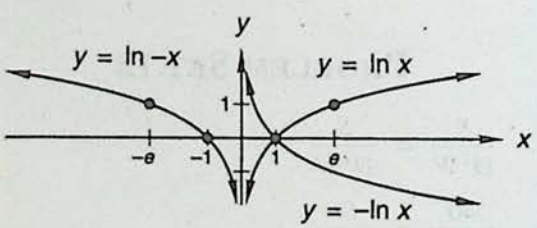


10.
$$\frac{\sin(x + \Delta x) - \sin x}{\Delta x}$$
$$= \frac{\sin x \cos \Delta x + \cos x \sin \Delta x - \sin x}{\Delta x}$$
$$= \frac{\sin x \cos \Delta x - \sin x}{\Delta x} + \frac{\cos x \sin \Delta x}{\Delta x}$$
$$= \frac{\sin x (\cos \Delta x - 1)}{\Delta x} + \frac{\cos x \sin \Delta x}{\Delta x}$$
$$= \sin x \left(\frac{\cos \Delta x - 1}{\Delta x}\right) + \cos x \left(\frac{\sin \Delta x}{\Delta x}\right)$$

11. For the key trigonometric identities, see section 12.A in the textbook.

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$
$$\cos(A + A) = \cos A \cos A - \sin A \sin A$$
$$\mathbf{\cos(2A) = \cos^2 A - \sin^2 A}$$
$$\cos(2A) = \cos^2 A - (1 - \cos^2 A)$$
$$\mathbf{\cos(2A) = 2\cos^2 A - 1}$$
$$\cos(2A) = (1 - \sin^2 A) - \sin^2 A$$
$$\mathbf{\cos(2A) = 1 - 2\sin^2 A}$$

12. (a) $\tan(A + B) = \dfrac{\sin(A + B)}{\cos(A + B)}$

$$\tan(A + B) = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$$
$$\tan(A + B) = \frac{\dfrac{\sin A \cos B}{\cos A \cos B} + \dfrac{\cos A \sin B}{\cos A \cos B}}{\dfrac{\cos A \cos B}{\cos A \cos B} - \dfrac{\sin A \sin B}{\cos A \cos B}}$$
$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

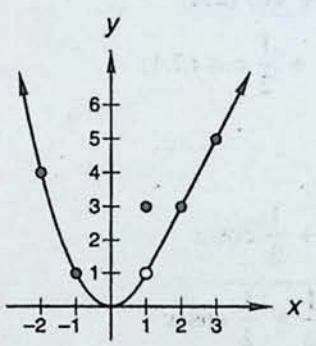
(b) $\tan(2A) = \tan(A + A)$

$$= \frac{\tan A + \tan A}{1 - \tan A \tan A} = \frac{2\tan A}{1 - \tan^2 A}$$

Since $\tan A = \dfrac{1}{2}$

$$\tan(2A) = \frac{2\left(\frac{1}{2}\right)}{1 - \left(\frac{1}{2}\right)^2} = \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}$$

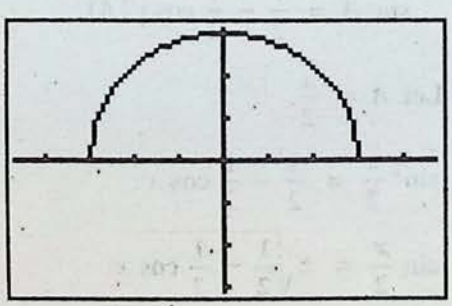
13.



14. (a) $\lim_{x \to 1^+} f(x) = 1$ (b) $\lim_{x \to 1^-} f(x) = 1$

(c) $f(1) = 3$

15. (a)



(b) $y = \sqrt{9 - x^2}$ describes only the positive square root, which coincides with the portions of the graph of a circle of radius 3 that lie on or above the x-axis.

(c) To graph a complete circle on a graphing calculator we need to graph it in two parts:

Y1=√(9−X²) and Y2=−√(9−X²)

16. The axis of symmetry is halfway between the roots.

$$x = \frac{2 + (-6)}{2} = -2$$

17. $A = \pi r^2$

$4\pi = \pi r^2$

$r = 2$

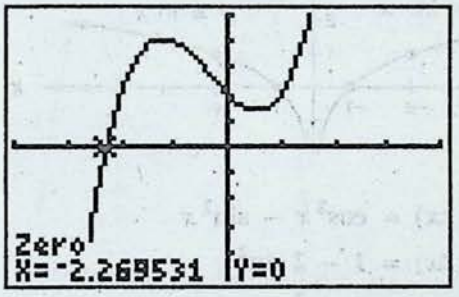
$(x - 1)^2 + (y + 2)^2 = 2^2$

18. $$\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{\dfrac{2}{x + \Delta x} - \dfrac{2}{x}}{\Delta x}$$

$$= \frac{\dfrac{2x - 2(x + \Delta x)}{x(x + \Delta x)}}{\Delta x} = \frac{\dfrac{-2\Delta x}{x(x + \Delta x)}}{\Delta x}$$

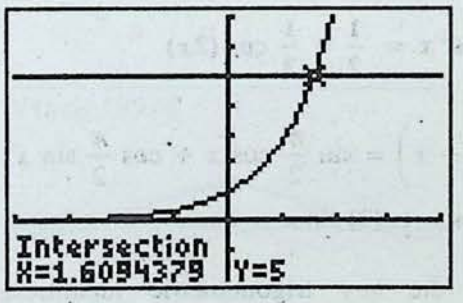
$$= \frac{-2}{x(x + \Delta x)}$$

19.



$x \approx -2.269531$

20. Let $Y_1=5$ and $Y_2=e^{\wedge}(X)$.



(a) **(1.609, 5)**

(b) $e^x = 5$

$x = \ln 5$

Point of intersection: **(ln 5, 5)**

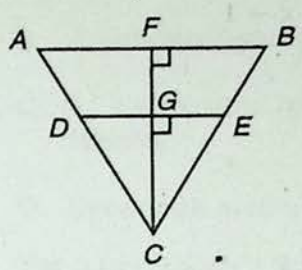
21. $y = \dfrac{\sqrt{x - 2}}{x}$

$x - 2 \geq 0$

$x \geq 2$ and division by zero is not allowed.

$\{x \in \mathbb{R} \mid x \geq 2\}$

22. Draw an auxillary line segment.



ΔCEG is a 30-60-90 right triangle.

Since $GC = h$, $GE = \dfrac{h}{\sqrt{3}}$ and $DE = \dfrac{2h}{\sqrt{3}}$

$$DE = \frac{2\sqrt{3}h}{3} \approx \mathbf{1.1547}h$$

23. $$\frac{1}{4}\sum_{n=1}^{4} f(1) = \frac{1}{4}[f(1) + f(1) + f(1) + f(1)]$$

$$= \frac{1}{4}(1 + 1 + 1 + 1) = \mathbf{1}$$

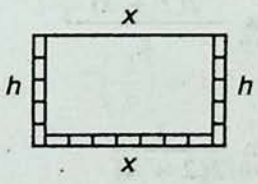
24. Since side CB is opposite $\angle A$ it must be greater than side AC which is opposite $\angle B$.

Quantity A is greater: **A**

25. $x^4 - y^4 = (x^2 + y^2)(x^2 - y^2) = 3(4) = \mathbf{12}$

Problem Set 14

1.



$xh = 100$

$h = \dfrac{100}{x}$

Perimeter $= 2x + 2h$

Cost $= 50(2h) + 50x + 20x$

$C = 100h + 70x$

$$C = 100\left(\frac{100}{x}\right) + 70x$$

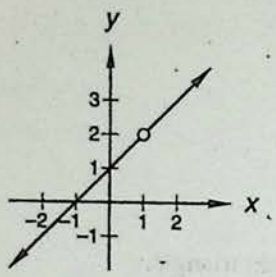
$$C = \mathbf{10{,}000}x^{-1} + \mathbf{70}x$$

2. $d = mh = rt$

$mh = r(h - 2)$

$$r = \frac{mh}{h - 2} \textbf{ miles per hour}$$

3. $y = \dfrac{x^2 - 1}{x - 1} = \dfrac{(x + 1)(x - 1)}{x - 1}$



4. $\lim\limits_{x \to 3} \dfrac{x^2 + 2x}{x + 2} = \dfrac{9 + 6}{5} = \dfrac{15}{5} = \mathbf{3}$

5. $\lim\limits_{x \to 2} \dfrac{x^2 + x - 6}{x - 2} = \lim\limits_{x \to 2} \dfrac{(x + 3)(x - 2)}{x - 2}$

$= \lim\limits_{x \to 2} (x + 3) = 2 + 3 = \mathbf{5}$

6. $\lim\limits_{x \to a} \dfrac{x^2 - a^2}{x - a} = \lim\limits_{x \to a} \dfrac{(x + a)(x - a)}{x - a}$

$= \lim\limits_{x \to a} (x + a) = a + a = \mathbf{2a}$

7. $\lim\limits_{x \to 0} \dfrac{(2 + x)^2 - 2^2}{x} = \lim\limits_{x \to 0} \dfrac{4 + 4x + x^2 - 4}{x}$

$= \lim\limits_{x \to 0} \dfrac{4x + x^2}{x} = \lim\limits_{x \to 0} (4 + x) = 4 + 0 = \mathbf{4}$

8. $\lim\limits_{x \to 0} \dfrac{\dfrac{1}{2 + x} - \dfrac{1}{2}}{x} = \lim\limits_{x \to 0} \dfrac{\dfrac{2 - (2 + x)}{2(2 + x)}}{x}$

$= \lim\limits_{x \to 0} \dfrac{\dfrac{-x}{2(2 + x)}}{x} = \lim\limits_{x \to 0} \dfrac{-1}{2(2 + x)}$

$= \dfrac{-1}{2(2 + 0)} = -\mathbf{\dfrac{1}{4}}$

9. $2 \sin^2 x - 3 \cos x = 3$

$2(1 - \cos^2 x) - 3 \cos x = 3$

$2 \cos^2 x + 3 \cos x + 1 = 0$

$(2 \cos x + 1)(\cos x + 1) = 0$

$2 \cos x + 1 = 0$ or $\cos x + 1 = 0$

$\cos x = -\dfrac{1}{2}$ $\qquad$ $\cos x = -1$

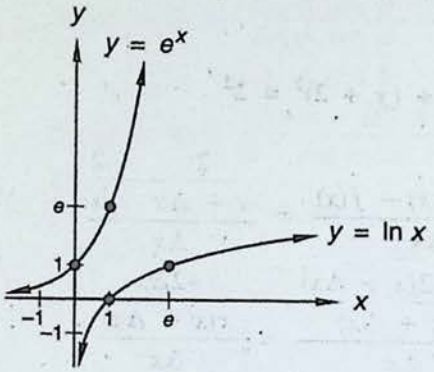
$x = \mathbf{120°, 180°, 240°}$

10. $y = 4 - 2 \sin (3x)$

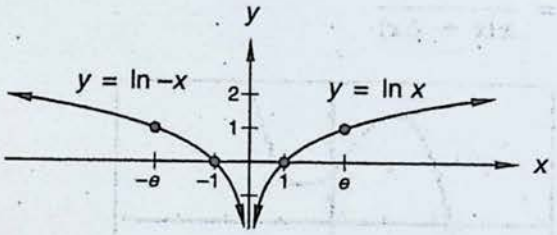
Amplitude: 2 $\qquad$ **Period:** $\dfrac{2\pi}{3}$

Centerline: $y = 4$

11.



12.



13. (a) $\cos (2x) = \cos^2 x - \sin^2 x$

$\cos (2x) = 1 - 2 \sin^2 x$

$\cos (2x) = 2 \cos^2 x - 1$

(b) $\cos (2x) = 2 \cos^2 x - 1$

$2 \cos^2 x = 1 + \cos (2x)$

$\cos^2 x = \dfrac{1}{2} + \dfrac{1}{2} \cos (2x)$

14. $\sin \left(\dfrac{\pi}{2} + x \right) = \sin \dfrac{\pi}{2} \cos x + \cos \dfrac{\pi}{2} \sin x$

$= (1) \cos x + (0) \sin x = \mathbf{\cos x}$

15. (a) For the key trigonometric identities, see section 12.A in the textbook.

$$\tan (A - B) = \frac{\sin (A - B)}{\cos (A - B)}$$

$$\tan (A - B) = \frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B + \sin A \sin B}$$

$$\tan (A - B) = \frac{\dfrac{\sin A \cos B}{\cos A \cos B} - \dfrac{\cos A \sin B}{\cos A \cos B}}{\dfrac{\cos A \cos B}{\cos A \cos B} + \dfrac{\sin A \sin B}{\cos A \cos B}}$$

$$\mathbf{\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}}$$

(b) $\tan 15° = \tan(60 - 45)°$

$$= \frac{\tan 60° - \tan 45°}{1 + \tan 60° \tan 45°} = \frac{\sqrt{3} - 1}{1 + \sqrt{3}(1)}$$

$$= \frac{\sqrt{3} - 1}{1 + \sqrt{3}}\left(\frac{1 - \sqrt{3}}{1 - \sqrt{3}}\right) = \frac{\sqrt{3} - 3 - 1 + \sqrt{3}}{1 - 3}$$

$$= \frac{-4 + 2\sqrt{3}}{-2} = 2 - \sqrt{3}$$

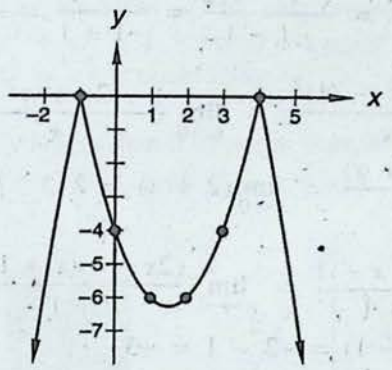
16. $y - 1 = \ln x$

$e^{y-1} = e^{\ln x}$

$x = e^{y-1}$

17. $y = -|x^2 - 3x - 4|$

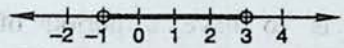
$y = -|(x - 4)(x + 1)|$



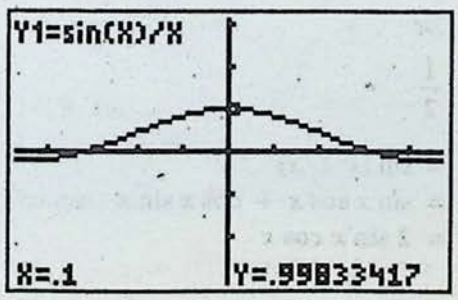
18. Choices B, C, and D will fail the vertical line test.

Only choice **A** is correct.

19. $|x - 1| < 2$ "x is less than 2 away from 1"



20.



Guess: $\lim_{x \to 0} \frac{\sin x}{x} = 1$

21. $(\sec -x)\left[\sin\left(\frac{\pi}{2} - x\right)\right]$

$+ (\sin -x)\left[\cos\left(\frac{\pi}{2} - x\right)\right]$

$= \sec x \cos x + (-\sin x) \sin x$

$= 1 - \sin^2 x = \cos^2 x$

22. A. A circle is not a function.

B. A vertically opening parabola IS a function.

C. A horizontally opening parabola is not a function.

D. Same graph as choice C.

The correct choice is **B.**

23. $\left(\frac{\sqrt{x+h} - \sqrt{x}}{h}\right)\left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}\right)$

$$= \frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

24. $\sum_{x=1}^{4} \frac{1}{x} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{25}{12}$

25. Regardless of whether x is positive, negative, or zero, $\sqrt{x^2} = |x|$.

The quantities are equal: **C**

Problem Set 15

1. $R_1 = \frac{1}{2}$; $R_2 = \frac{1}{6}$

$T(R_1 + R_2) = 1$

$T\left(\frac{1}{2} + \frac{1}{6}\right) = 1$

$T\left(\frac{2}{3}\right) = 1$

$T = \frac{3}{2}$ **hr**

2. $S + L = 40$

$S = 40 - L$

$P = SL$

$\boldsymbol{P = 40L - L^2}$

3. (a) $(-\infty, -2)$ and $(2, \infty)$

(b) $(-2, 2)$

(c) $(0, \infty)$

(d) $(-\infty, 0)$

4. Possible rational roots: $\pm 1, \pm 2, \pm 4$

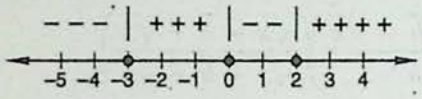
$$\begin{array}{c|cccc} -2 & 1 & 0 & -6 & -4 \\ & \downarrow & -2 & 4 & 4 \\ \hline & 1 & -2 & -2 & \boxed{0} \end{array}$$

$$x^3 - 6x - 4 = 0$$

$$(x + 2)(x^2 - 2x - 2) = 0$$

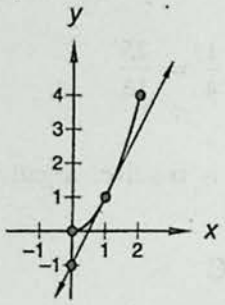
$$x = -2, 1 \pm \sqrt{3}$$

5.



Zeros: **–3, 0, 2**

6.



Answers may vary: (1, 1), (0, –1)

$$\text{slope} \approx \frac{1 - (-1)}{1 - 0} = \frac{2}{1} = 2$$

7. $\tan^2 x = 1$

$\tan x = \pm 1$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

8. $$\sin^2 x - \sin x + \frac{1}{4} = 0$$

$$\left(\sin x - \frac{1}{2}\right)\left(\sin x - \frac{1}{2}\right) = 0$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

9. $\arcsin x = y$

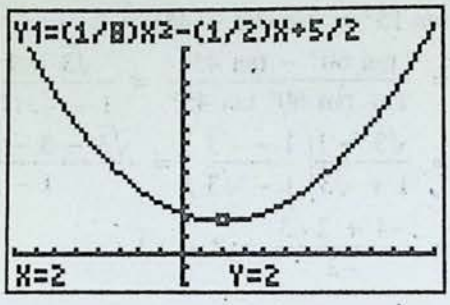
$x = \sin y$

10. (a) $x^2 - 8y - 4x + 20 = 0$

$$8y = x^2 - 4x + 20$$

$$y = \frac{1}{8}x^2 - \frac{1}{2}x + \frac{5}{2}$$

(b)



(c) **(2, 2)**

11. $$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1} \frac{(x + 1)(x - 1)}{x - 1}$$

$$= \lim_{x \to 1} (x + 1) = 1 + 1 = 2$$

12. $$\lim_{x \to -1} \frac{x^2 + 1}{x - 1} = \frac{(-1)^2 + 1}{-1 - 1} = \frac{1 + 1}{-1 - 1} = -1$$

13. $$\lim_{x \to 0} \frac{(1 + x)^2 - (1)^2}{x} = \lim_{x \to 0} \frac{1 + 2x + x^2 - 1}{x}$$

$$= \lim_{x \to 0} \frac{x(2 + x)}{x} = \lim_{x \to 0} (2 + x) = 2$$

14. $$\lim_{x \to -1} \frac{2x^2 + x - 1}{x + 1} = \lim_{x \to -1} \frac{(2x - 1)(x + 1)}{x + 1}$$

$$= \lim_{x \to -1} (2x - 1) = -2 - 1 = -3$$

15. $$\frac{[2(x + \Delta x) + 3] - (2x + 3)}{\Delta x}$$

$$= \frac{2x + 2(\Delta x) + 3 - 2x - 3}{\Delta x} = \frac{2(\Delta x)}{\Delta x} = 2$$

16. If $y = \frac{1}{k} \sin(kx)$ is to have a period of 4π, then $\frac{2\pi}{k} = 4\pi$.

$$4\pi(k) = 2\pi$$

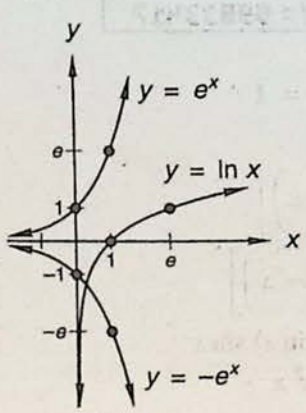
$$k = \frac{1}{2}$$

17. $\sin(2x) = \sin(x + x)$

$\sin(2x) = \sin x \cos x + \cos x \sin x$

$\sin(2x) = 2 \sin x \cos x$

18.



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