Algebra $\frac{1}{2}$

An Incremental Development

THIRD EDITION

SAXON

Contents

This book will prove to you that mathematics is reasonable and that mathematics is not hard. If you do every problem in every problem set, you will be amazed at how easy it will all become. Let me say again: algebra is not **difficult.** Algebra is just **different.** Things that are different become familiar only after you have been exposed to them for a long time.

acknowledgments In closing, I would like to thank the staff who contributed to the revision of this textbook. In particular, I thank editors Brian Rice, Matt Maloney, and Clint Keele; copy editors Chris Davey and Jenifer Sparks; accuracy checkers Karen Bottoms, Kara Chiodo, and Julia Suggs; production team leader Carrie Brown; production team members Sariah Adams, Eric Atkins, Wendy Chitwood, Jane Claunch, David LeBlanc, Chad Morris, Tonea Morrow, Dan O'Conner, Nancy Rimassa, Debra Sullivan, Jean Van Vleck, Sunnie Wagner, and Loren White; and production coordinator Angela Johnson. I also thank Travis Southern, who led Jeremy Eiken, Larry Gilchrist, Sherri Little, and Erin McCain in the editorial work during its formative stages. Finally, I thank Julie Webster, editor-in-chief, and Travis Rose, production manager, for their leadership of the editorial and production departments during the project.

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LESSON 1 Whole Number Place Value • Expanded Notation • Reading and Writing Whole Numbers • Addition

1.A whole number place value

We use the **Hindu-Arabic system** to write our numbers. This system is a base 10 system and thus has ten different symbols. The symbols are called **digits,** or **numerals,** and they are

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

The numbers we say when we count are called **counting numbers,** or **natural numbers.** We may show the set of counting numbers this way:

{1, 2, 3, 4, 5, ...}

The three dots, called an *ellipsis*, mean that the list continues without end. The symbols, { }, are called braces and are sometimes used to designate a set. If we include zero with the set of counting numbers, then we form the set of **whole numbers.**

{0, 1, 2, 3, 4, ...}

When we write whole numbers, we can write the **decimal point** at the end of the number, or we can leave it off. Both of these

427.   427

represent the same number. In the right-hand number, the decimal point is assumed to be after the 7.

The value of a digit in a number depends on where the digit appears in the number. The first place to the left of the decimal point is the ones' place. We also call this place the **units' place,** which has a **place value** of 1. The next place to the left of the units' place is the **tens' place,** with a place value of 10, followed by the **hundreds' place,** with a place value of 100, and then the **thousands' place,** with a place value of 1,000. Each place to the left has one more zero.

Whole Number Place Values

100,000,000,000,000	hundred trillions
10,000,000,000,000	ten trillions
1,000,000,000,000	trillions
,	
100,000,000,000	hundred billions
10,000,000,000	ten billions
1,000,000,000	billions
,	
100,000,000	hundred millions
10,000,000	ten millions
1,000,000	millions
,	
100,000	hundred thousands
10,000	ten thousands
1,000	thousands
,	
100	hundreds
10	tens
1	units
.	decimal point

To find the value of a digit in a number, multiply the digit times the place value. For example, the 5 in the left-hand number below

415,623   701,586   731,235

has a value of 5×1000, or 5000, because it is in the thousands' place. The value of the 5 in the center number is 5×100, or 500, because it is in the hundreds' place. The value of the 5 in the right-hand number is 5×1, or 5, because it is in the units' (ones') place.

example 1.1 In the number 46,235:

(a) What is the value of the digit 5?

(b) What is the value of the digit 2?

(c) What is the value of the digit 4?

solution First we write the decimal point at the end of the number.

46,235.

(a) The 5 is one place to the left of the decimal point. This is the units' place. This digit has a value of 5×1, or **5.**

(b) The 2 is three places to the left of the decimal point. This is the hundreds' place. This digit has a value of 2×100, or **200.**

(c) The 4 is five places to the left of the decimal point. This is the ten-thousands' place. This digit has a value of $4 \times 10{,}000$, or **40,000.**

1.B expanded notation

Writing a number in **expanded notation** is a good way to practice the idea of place value. When we write a number in expanded notation, we consider the value of every digit in the number individually. To write a number in expanded notation, we write each of the nonzero digits multiplied by the place value of the digit. We use parentheses to enclose each of these multiplications and put a plus sign between each set of parentheses.

To write 5020 in expanded notation, we write

$$(5 \times 1000) + (2 \times 10)$$

because this number contains five thousands and two tens.

example 1.2 Write the following number in standard notation: $(4 \times 10{,}000) + (6 \times 100) + (5 \times 1)$

solution Standard notation is our usual way of writing numbers. The number has four ten thousands, no thousands, six hundreds, no tens, and five ones. The number is **40,605.**

example 1.3 Write the number 6,305,126 in expanded notation.

solution There are six millions, $(6 \times 1{,}000{,}000)$ three hundred thousands, $(3 \times 100{,}000)$ five thousands, (5×1000) one hundred, (1×100) two tens, (2×10) and six ones. (6×1)

If we add them all together, we get

$$\mathbf{(6 \times 1{,}000{,}000) + (3 \times 100{,}000) + (5 \times 1000) + (1 \times 100) + (2 \times 10) + (6 \times 1)}$$

1.C reading and writing whole numbers

We begin by noting that all numbers between 20 and 100 that do not end in zero are hyphenated words when we write them out.

23 is written twenty-three	64 is written sixty-four
35 is written thirty-five	79 is written seventy-nine
42 is written forty-two	86 is written eighty-six
51 is written fifty-one	98 is written ninety-eight

The hyphen is also used in whole numbers when the whole number is used as a modifier. The words

ten thousand

are not hyphenated. But when we use these words as a modifier, as when we say

ten-thousands' place,

the words are hyphenated. Other examples of this rule are

hundred-millions' digit
ten-billions' place
hundred-thousands' place

The word *and* is not used when we write out whole numbers.

501	is written	five hundred one
	not	five hundred and one
370	is written	three hundred seventy
	not	three hundred and seventy
422	is written	four hundred twenty-two
	not	four hundred and twenty-two

Do not think of this as a useless exercise! Knowing how to correctly and accurately write numbers is necessary when writing a check, for example. Before we read whole numbers, we place a comma after every third digit beginning at the decimal point and moving to the left.† The commas divide the numbers into groups of three digits.

Place Value

Trillions			Billions			Millions			Thousands			Units (Ones)			
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	· Decimal point

To read the number 4125678942, we begin on the right-hand end, write a decimal point, and separate the number into groups of three by writing commas.

4,125,678,942.

Then we read the number, beginning with the leftmost group. First we read the number in the group, and then we read the name of the group. Then we move to the right and repeat the procedure.

four billion, one hundred twenty-five million, six hundred seventy-eight thousand, nine hundred forty-two

†It is our convention to usually write four-digit whole numbers without commas.

example 1.4 Use words to write this number: 51723642

solution We write the decimal point on the right-hand end. Then we move to the left and place a comma after each group of three digits.

51,723,642.

The leftmost group is the millions' group. We read it as

fifty-one million,

and write the comma after the word *million*. The next three-digit group is the thousands' group. We read it as

seven hundred twenty-three thousand,

and we write the comma after the word *thousand*. The last three-digit group is the units' group. We do not say "units" but just read the three-digit number as

six hundred forty-two

Note that the words *fifty-one*, *twenty-three*, and *forty-two* are hyphenated. Also note that we do not use the word *and* between the groups. Now we put the parts together and read the number as

fifty-one million, seven hundred twenty-three thousand, six hundred forty-two

Note that the commas appear in the same places the commas appeared when we used digits to write the number.

example 1.5 Use digits to write the number fifty-one billion, twenty-seven thousand, five hundred twenty.

solution The first group is the billions' group. It contains the number fifty-one.

51, , ,

All the groups after the first group must have three digits. There are no millions, so we use three 0's.

51,000, ,

There are twenty-seven thousands. We write 027 in the next group so that the group will contain three digits.

51,000,027,

Now we finish by writing 520 in the last group.

51,000,027,520

1.D addition

When we add numbers, we call each of the numbers **addends,** and we call the answer a **sum.**

523	addend
619	addend
+ 512	addend
1654	sum

To add whole numbers, we write the numbers so that the units' places of the numbers are aligned vertically. Then we add in columns.

example 1.6 Add: $4 + 407 + 3526$

solution We write the numbers so that the units' places of the numbers are aligned vertically. Then we add.

$$\begin{array}{r} 4 \\ 407 \\ +\ 3526 \\ \hline \mathbf{3937} \end{array}$$

To add money, we write each amount of money with a dollar sign, and with two places to the right of the decimal point. Then we align the decimal points and add.

example 1.7 Add: $\$2.54 + \$5 + 9¢$

solution We write the numbers so that there are two places to the right of each decimal point and align the decimal points. We include dollar signs, and then we add.

$$\begin{array}{r} \$2.54 \\ \$5.00 \\ +\ \$0.09 \\ \hline \mathbf{\$7.63} \end{array}$$

This book is designed to permit the reader to automate the upper-level skills of arithmetic while the concepts of algebra are being introduced. The addition, subtraction, multiplication, and division problems in the practice and problem sets are designed to provide paper-and-pencil practice in the four basic operations of arithmetic. **Throughout this book, do not use a calculator unless you are instructed to do so by your teacher.**

practice

a. In the number 152068, what is the value of the 2?

b. Write in standard notation: $(6 \times 1000) + (4 \times 10) + (3 \times 1)$

c. Write 85,020 in expanded notation.

d. Use digits to write this number: ten billion, two hundred five million, forty-one thousand, five hundred

e. Use words to write this number: 36025103

problem set 1

1. In the number 5062973, what is the value of each of these digits?
(a) 6 (b) 9 (c) 3

2. Write the six-digit number that has the digit 4 in the thousands' place, with each of the remaining digits being 3.

3. Write the seven-digit number that has the digit 3 in the millions' place and the digit 7 in the hundreds' place, with each of the remaining digits being 6.

4. A number has eight digits. Every digit is 9 except the ten-millions' digit, which is 3, the ten-thousands' digit, which is 5, and the units' digit, which is 2. Use digits to write the number.

5. Use digits to write this number: forty-one billion, two hundred thousand, five hundred twenty

6. Use digits to write this number: five hundred seven billion, six hundred forty million, ninety thousand, forty-two

7. Use digits to write this number: four hundred seven trillion, ninety million, seven hundred forty-two thousand, seventy-two

8. Use digits to write this number: nine hundred eighty million, four hundred seventy

Use words to write each number:

9. 517236428

10. 90807060

11. 32000000652

12. 3250009111

13. 6040000

14. 99019900

Write each number in standard notation:

15. $(3 \times 100{,}000) + (4 \times 1000) + (2 \times 10)$

16. $(7 \times 10{,}000) + (8 \times 100) + (6 \times 10)$

17. $(9 \times 1000) + (4 \times 100) + (5 \times 1)$

18. $(7 \times 1{,}000{,}000) + (2 \times 10{,}000) + (6 \times 1000)$

Write each number in expanded notation:

19. 5280

20. 408

21. 70,600

22. 21,000

23. 4005

24. 9080

Add:

25.
$$\begin{array}{r} 43 \\ 76 \\ 84 \\ +\ 91 \\ \hline \end{array}$$

26.
$$\begin{array}{r} 4628 \\ 5734 \\ +\ 8416 \\ \hline \end{array}$$

27.
$$\begin{array}{r} \$53.58 \\ +\ \$52.78 \\ \hline \end{array}$$

28. $9056 + 4708 + 9076$

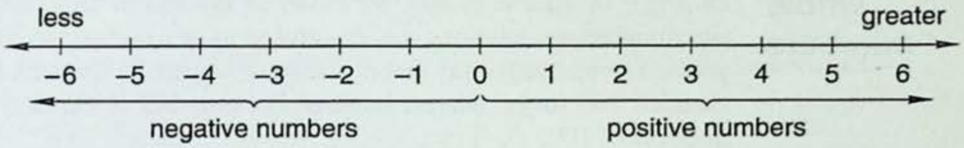
29. $432 + 846 + 943 + 721$

30. $\$3.64 + 52¢ + \$9$

LESSON 2 *The Number Line and Ordering • Rounding Whole Numbers*

2.A the number line and ordering

A **number line** can be used to help us arrange numbers in order. Each number corresponds to a unique point on the number line. The zero point of a number line is called the **origin.** The numbers to the right of the origin are called **positive numbers,** and they are all greater than zero. Every positive number has an **opposite** that is the same distance to the left of the origin. The numbers to the left of the origin are called **negative numbers.** The negative numbers are all less than zero. Zero is neither positive nor negative.



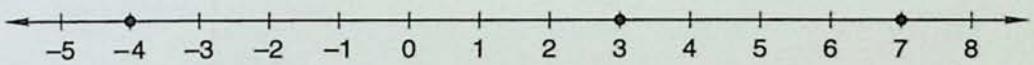
On this number line, the tick marks indicate the location of **integers.** The set of integers includes all of the counting numbers as well as their opposites and the number zero.

$$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

Notice that the negative numbers are written with a negative sign. For −5 we say "negative five." Positive numbers may be written with or without a positive sign. Both +5 and 5 are positive and equal to each other.

As we move to the right on a number line, the numbers become greater and greater. As we move to the left on a number line, the numbers become less and less. A number is greater than another number if it is farther to the right on a number line.

When we put a dot on a number line to mark the location of a number, we call the dot the **graph** of the number. The number associated with a point on the number line is called the **coordinate** of the point. Any group of numbers can be arranged in order from the least to the greatest. If we graph the numbers 3, −4, and 7 on a number line, we get this figure:

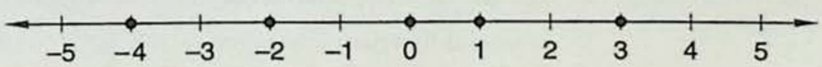


When we graph these numbers, we see that the least number is on the left and the greatest number is on the right.

example 2.1 Use a number line to arrange these integers in order from least to greatest:

$$3, -4, 0, 1, -2$$

solution To arrange these numbers in order, we first graph each of the numbers on a number line.



We arrange the numbers in the order they appear from left to right on the number line.

$$\mathbf{-4, -2, 0, 1, 3}$$

example 2.2 Arrange the following numbers in order from least to greatest: 465, 654, −546, 456, and 564

solution Negative numbers are always less than positive numbers, so −546 is the least number. Since all the positive numbers have three digits, the number with the least value among these is the one with the digit of least value in the hundreds' place. We have 2 numbers that begin with 4:

465 and 456

Now we look at the second digits (those in the tens' place) and see that 456 is the lesser of the two numbers because 5 is less than 6. So the two positive numbers with the least value are, in order,

456 and 465

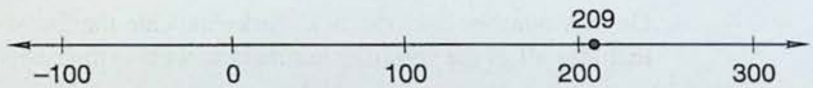
This leaves 654 and 564. Of course, 564 is less than 654 because it has the lesser value in the hundreds' place. So our answer is

–546, 456, 465, 564, 654

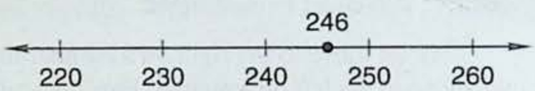
If we graphed these numbers on a number line, this is the order in which they would appear.

2.B rounding whole numbers

We often use the **rounded** form of a number. We round because some numbers, such as multiples of 10 and of 100, are easier to say and to think about than others, and in many situations approximations are as useful as more exact values. If the distance to a barn is 209 yards, a farmer might say that it is about 200 yards to the barn. If so, we could say the farmer rounded 209 to the nearest hundred because 209 is closer to 200 than it is to any other multiple of 100.



If it is 246 yards to the barn, the farmer would say that it is about 250 yards to the barn. In this case, we could say the farmer rounded 246 to the nearest 10 because 246 is closer to 250 than it is to any other multiple of 10.



Rather than draw a number line, we will use a circle and an arrow to help us round numbers. To demonstrate, we will round 24,374 to the nearest thousand using three steps.

1. Circle the digit in the place to which we are rounding and mark the digit to its right with an arrow.

   ↓
   2④,374

2. Leave the circled digit unchanged or increase it one unit as determined by the following rules:
   (a) If the arrow-marked digit is less than 5, do not change the circled digit.
   (b) If the arrow-marked digit is 5 or greater, increase the circled digit by one unit.

   Rule (a) applies in this problem, so we leave the circled digit unchanged.
3. Change the arrow-marked digit and all digits to its right to zero.

   ↓
   2④,000

   Our answer is 24,000.

example 2.3 Round 471,326,502 to the nearest ten thousand.

solution First we circle the ten-thousands' digit and mark the digit to its right with an arrow.

↓
471,3②6,502

Since the arrow-marked digit is greater than 5, we increase the circled digit from 2 to 3.

↓
471,3③6,502

Next change the arrow-marked digit and all digits to its right to zero.

↓
471,3③0,000

So our answer is **471,330,000.**

example 2.4 Round 83,752,914,625 to the nearest ten million.

solution First we circle the ten-millions' digit and mark the digit to its right with an arrow.

↓
83,7⑤2,914,625

Since the arrow-marked digit is less than 5, we leave the circled digit unchanged. Now we change the arrow-marked digit and all digits to its right to zero.

83,750,000,000

practice

a. Use a number line to arrange these numbers in order: –1, 4, –7, 0, 7, 5

b. Arrange the following numbers in order from least to greatest: 736, –637, 367, 376, 673

c. Round 914,471,752 to the nearest ten thousand.

d. Round 83,625,502 to the nearest thousand.

problem set 2

†**1.** *(2)* Use a number line to arrange these numbers in order from least to greatest: 2, 19, –11, 6, –5

Arrange the following numbers in order from least to greatest:

2. *(2)* 514, 154, 145, –415, 451

3. *(2)* 942, 249, 924, 294, 429

4. *(2)* Round 4,185,270 to the nearest hundred.

5. *(2)* Round 83,721,525 to the nearest thousand.

6. *(2)* Round 415,237,842 to the nearest hundred thousand.

7. *(1)* A number has nine digits. All the digits are 7 except the ten-thousands' digit, which is 2, and the tens' digit, which is 5. Write the number.

8. *(1)* A number has seven digits. All the digits are 3 except the hundred-thousands' digit, which is 6, the thousands' digit, which is 4, and the hundreds' digit, which is 7. Write the number.

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†The italicized numbers within parentheses below each problem number refer to the lesson in which the concepts for that problem are discussed.

example 3.3 Find the missing number:

$$\begin{array}{r} 472 \\ +\ A \\ \hline 628 \end{array}$$

solution The missing number is an addend, the difference between 628 and 472. **This is an addition pattern, but we need to subtract to find the missing number.**

$$\begin{array}{r} \overset{5}{\cancel{6}}\,{}^{1}2\,8 \\ -\ 4\,7\,2 \\ \hline 1\,5\,6 \end{array}$$

Now we check by adding.

$$\begin{array}{r} 472 \\ +\ 156 \\ \hline 628 \end{array} \quad \text{check}$$

The value of A is **156.**

example 3.4 Find the missing number: $M + 257 = 493$

solution What number added to 257 equals 493? To find the number, an addend, we will subtract 257 from 493.

$$\begin{array}{r} 4\,\overset{8}{\cancel{9}}\,{}^{1}3 \\ -\ 2\,5\,7 \\ \hline 2\,3\,6 \end{array}$$

Now we check.

$$\begin{array}{r} 236 \\ +\ 257 \\ \hline 493 \end{array} \quad \text{check}$$

The value of M is **236.**

Consider the pattern of the following subtraction problem:

$$\begin{array}{rl} 822 & \text{minuend—the greatest number} \\ -\ 357 & \text{subtrahend} \\ \hline 465 & \text{difference} \end{array}$$

Again three numbers are involved. When we know two of them, we can find the third number. If the difference is missing, we subtract; if the subtrahend is missing, we subtract the difference from the minuend; and if the minuend is missing, we add the subtrahend and difference.

example 3.5 Find the missing number:

$$\begin{array}{r} 526 \\ -\ X \\ \hline 329 \end{array}$$

solution To find X, the subtrahend, we subtract 329 from 526.

$$\begin{array}{r} \overset{4}{\cancel{5}}\,{}^{1}\overset{1}{\cancel{2}}\,{}^{1}6 \\ -\ 3\,2\,9 \\ \hline 1\,9\,7 \end{array}$$

Now we check using our subtraction pattern.

$$\begin{array}{r} 526 \\ -\ 197 \\ \hline 329 \end{array} \quad \text{check}$$

The value of X is **197.**

example 3.6 Find the missing number: $P - 423 = 287$

solution The first number in a subtraction pattern, the minuend, always equals the sum of the subtrahend and the difference. So we add the subtrahend and difference to find P.

$$\begin{array}{r} 423 \\ +\ 287 \\ \hline 710 \end{array}$$

Now we check using our subtraction pattern.

$$\begin{array}{r} 710 \\ -\ 423 \\ \hline 287 \end{array} \quad \text{check}$$

The value of P is **710.**

practice

a. Subtract: $54.85 − $27.68

Find the missing numbers:

b. $\begin{array}{r} 563 \\ +\ A \\ \hline 912 \end{array}$

c. $K + 123 = 522$

d. $\begin{array}{r} M \\ -\ 364 \\ \hline 376 \end{array}$

e. $741 - F = 372$

problem set 3

1. (1) A number has six digits. Every digit is a 2 except the thousands' digit, which is 5, and the units' digit, which is 3. What is the number?

2. (1) A number has five digits. Every digit is a 7 except the thousands' digit, which is 0. What is the number?

3. (1) A number has seven digits. Every digit is a 4 except the hundred-thousands' digit, which is 1. What is the number?

4. (1) Use digits to write fourteen million, seven hundred five thousand, fifty-two.

5. (1) Use digits to write five hundred billion, four hundred sixty-five thousand, one hundred eighty-two.

Write in expanded notation:

6. (1) 64,030

7. (1) 79,003

8. (1) 123,419

Add or subtract as indicated:

9. (3) $\begin{array}{r} 551 \\ -\ 174 \\ \hline \end{array}$

10. (3) $\begin{array}{r} 853 \\ -\ 284 \\ \hline \end{array}$

11. (1) $\begin{array}{r} 936 \\ +\ 474 \\ \hline \end{array}$

12. (1) $\begin{array}{r} 839 \\ +\ 472 \\ \hline \end{array}$

13. (3) $60 − $49.49

14. (3) 4017 − 3952

Find the missing numbers:

15. (3)
$$\begin{array}{r} X \\ -\ 245 \\ \hline 276 \end{array}$$

16. (3) $800 - M = 436$

17. (3)
$$\begin{array}{r} 735 \\ +\quad A \\ \hline 1211 \end{array}$$

18. (3) $925 + F = 1111$

Write in standard notation:

19. (1) $(8 \times 10{,}000) + (5 \times 1000) + (3 \times 100) + (2 \times 10) + (5 \times 1)$

20. (1) $(6 \times 1000) + (6 \times 100) + (6 \times 1)$

21. (1) $(3 \times 100{,}000) + (2 \times 10{,}000) + (9 \times 100) + (7 \times 1)$

22. (1) Use words to write 5803125702.

Add:

23. (1) $295 + 486 + 588 + 714$

24. (1) \$2 + \$9.37 + 86¢

25. (1)
$$\begin{array}{r} 90{,}125 \\ 40{,}061 \\ 30{,}627 \\ +\ 95{,}132 \\ \hline \end{array}$$

26. (1)
$$\begin{array}{r} 1125 \\ 986 \\ 139 \\ +\ 2364 \\ \hline \end{array}$$

27. (2) Round 716,487,250 to the nearest ten million.

28. (2) Round 716,487,250 to the nearest ten thousand.

29. (2) Use a number line to arrange these numbers from least to greatest: 6, –2, 1, 0, –4

30. (2) Arrange the following numbers in order from least to greatest:
321, –213, 123, 231, 132

LESSON 4 Multiplication • Division • Multiplication and Division Patterns

4.A multiplication

Multiplication is a shorthand notation we use to denote repeated addition. If we wish to add 7 twelve times, we could write

$$7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 = 84$$

or we could write

$$7 \times 12 = 84$$

which also means, "twelve sevens is eighty-four." If we wish to add 12 seven times, we could write

$$12 + 12 + 12 + 12 + 12 + 12 + 12 = 84$$

or we could write

$$12 \times 7 = 84$$

which is the more compact way of saying, "seven twelves is eighty-four." Because the sum of twelve 7's is the same as the sum of seven 12's, we could write for either addition problem

$$7 \times 12 \qquad \text{or} \qquad 12 \times 7$$

Here we use a cross to indicate multiplication. In algebra, we often use a center dot instead of a cross. The dot also indicates multiplication. We can write the multiplications above as

$$7 \cdot 12 \qquad \text{or} \qquad 12 \cdot 7$$

When we multiply two numbers, the order of the numbers does not affect the result.

In multiplication, we call numbers that are multiplied together **factors.** We call the answer the **product.**

$$\begin{array}{rl} 12 & \text{factor} \\ \times\ 7 & \text{factor} \\ \hline 84 & \text{product} \end{array}$$

example 4.1 Multiply: 164×23

solution We usually write the number with the most digits on top. We first multiply by the 3 of 23, then by the 20 of 23. We add the partial products and find the final product.

$$\begin{array}{r} 164 \\ \times\ 23 \\ \hline 492 \\ 328\ \\ \hline \mathbf{3772} \end{array}$$

example 4.2 Multiply: $\$5.79 \times 29$

solution We multiply as usual and then write the product with a dollar sign and two decimal places.

$$\begin{array}{r} \$5.79 \\ \times\quad 29 \\ \hline 52\ 11 \\ 115\ 8\ \ \\ \hline \mathbf{\$167.91} \end{array}$$

4.B division

When we divide, we divide the **dividend** by the **divisor.** The result of the division is the **quotient.** Division can be indicated by either a division sign (÷), a division box ($\overline{)\quad}$), or a division bar (–). Proper placement of the dividend and divisor must be maintained to arrive at the quotient.

$$\text{dividend} \div \text{divisor} = \text{quotient} \qquad \text{divisor}\overline{)\text{dividend}}^{\text{quotient}}$$

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient}$$

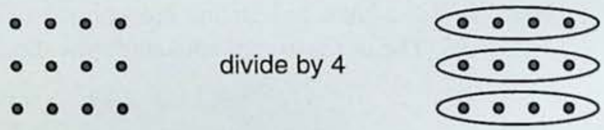
Here are three ways to indicate the division of 20 by 4.

$$20 \div 4 \qquad 4\overline{)20} \qquad \frac{20}{4}$$

It is sometimes helpful to think of division as a process of separating the dividend into a number of equal groups. For instance, if we wish to divide 12 by 4, we may write

$$\frac{12}{4}$$

The primary question we are asking is, "If we divide twelve objects into four equal groups, how many will be in each group?" Some people find it easier to visualize the equivalent question, "Into how many groups of four can we divide twelve objects?" We can display the solution visually by using twelve dots and arranging them in groups of four.



We see that twelve dots can be divided into three groups of four, so we may say that

$$\frac{12}{4} = 3$$

An **algorithm** is a step-by-step process to solve a particular type of problem. When mathematicians speak of a division algorithm, they are merely describing a procedure for solving a division problem.

example 4.3 Divide: $\frac{\$13.70}{5}$

solution We will use the common division algorithm. The quotient is written with a dollar sign, and the decimal point in the answer is directly above the decimal point in the dividend.

$$\begin{array}{r} \$2.74 \\ 5\overline{)\$13.70} \\ \underline{10} \\ 3\,7 \\ \underline{3\,5} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

Thus,

$$\frac{\$13.70}{5} = \mathbf{\$2.74}$$

example 4.4 Divide: $2183 \div 47$

solution We use the same algorithm.

$$\begin{array}{r l} 46 & \\ 47\overline{)2183} & \\ \underline{188} & \\ 303 & \\ \underline{282} & \\ 21 & \text{remainder} \end{array}$$

Thus,

$$2183 \div 47 = \mathbf{46\ r\ 21}$$

4.C multiplication and division patterns

Remember that addition and subtraction are inverse operations because either one of these operations will "undo" the other operation. **Multiplication and division are also inverse operations.** To demonstrate, we begin with the number 5, and then multiply by 4 to get 20.

$$5 \times 4 = 20$$

Now if we divide 20 by 4, we will undo the multiplication by 4 and be back at 5 again.

$$20 \div 4 = 5$$

Thus, multiplication and division are inverse operations.

Consider the pattern of the following multiplication problem:

$$\begin{array}{r l} 8 & \text{factor} \\ \underline{\times\ 9} & \text{factor} \\ 72 & \text{product} \end{array}$$

Three numbers are involved. When we know two of them, we can find the missing number. If the product is missing, we multiply the factors; but if a factor is missing, we find the missing factor by dividing the product by the known factor.

example 4.5 Find the missing number: $12 \cdot N = 168$

solution The missing number is one of the two factors, so we divide the product by the known factor.

$$\begin{array}{r} 14 \\ 12\overline{)168} \\ \underline{12} \\ 48 \\ \underline{48} \\ 0 \end{array}$$

Now we check by multiplying.

$$\begin{array}{r l} 12 & \\ \underline{\times\ 14} & \\ 48 & \\ \underline{12\ \ } & \\ 168 & \text{check} \end{array}$$

The value of N is **14**.

Consider the pattern of the following division problem:

$$\frac{56}{7} = 8 \quad \text{or} \quad \frac{\text{dividend}}{\text{divisor}} = \text{quotient}$$

Again three numbers are involved. When we know two of them, we can find the third number. If the quotient is missing, we divide; if the divisor is missing, we divide the dividend by the quotient; and if the dividend is missing, we multiply the divisor and quotient.

example 4.6 Find the missing number: $\frac{64}{A} = 4$

solution To find a missing divisor, we divide the dividend by the quotient.

$$\begin{array}{r} 16 \\ 4\overline{)64} \\ \underline{4} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

Now we check by dividing.

$$\begin{array}{r} 4 \\ 16\overline{)64} \\ \underline{64} \\ 0 \end{array} \quad \text{check}$$

The value of A is **16.**

example 4.7 Find the missing number: $B \div 3 = 15$

solution To find a missing dividend, we multiply the divisor and the quotient.

$$\begin{array}{r} 15 \\ \underline{\times \ \ 3} \\ 45 \end{array}$$

Now we check by dividing.

$$\begin{array}{r} 15 \\ 3\overline{)45} \\ \underline{3} \\ 15 \\ \underline{15} \\ 0 \end{array}$$

The value of B is **45.**

practice Multiply or divide as indicated:

a. $300 \times \$1.25$

b. $778 \cdot 563$

c. $\frac{261}{30}$

d. $12\overline{)\$49.08}$

Find the missing numbers:

e. $15 \cdot C = 180$

f. $\frac{x}{3} = 56$

g. $\frac{368}{B} = 92$

problem set 4

1. (1) A number has four digits. All the digits are 3 except for the thousands' digit, which is 7. What is the number?

2. (1) A number has seven digits. All the digits are 3 except for the hundreds' digit and the thousands' digit, both of which are 9. What is the number?

3. (1) Use digits to write forty-seven million, fourteen.

4. (1) Use digits to write fourteen billion, forty-two thousand, seven hundred fifty-five.

5. (2) Round 716,487,250 to the nearest million.

6. (2) Arrange the following numbers in order from least to greatest:
671, –76, 167, –176, 617

7. (1) Write 7650 in expanded notation.

8. (1) Write $(5 \times 1000) + (6 \times 10) + (7 \times 1)$ in standard notation.

Divide:

9. (4) $\dfrac{\$25.11}{9}$

10. (4) $2800 \div 50$

11. (4) $45\overline{)50,217}$

12. (4) $\dfrac{9114}{7}$

13. (4) $4165 \div 40$

14. (4) $21\overline{)30,215}$

Multiply:

15. (4) $\begin{array}{r} 285 \\ \times\ 321 \\ \hline \end{array}$

16. (4) $\begin{array}{r} \$5.06 \\ \times\ \ \ 75 \\ \hline \end{array}$

17. (4) $\begin{array}{r} 512 \\ \times\ 320 \\ \hline \end{array}$

18. (4) $25 \times 40 \times 100$

19. (4) 500×420

20. (4) $6 \times 12 \times 24$

Find the missing numbers:

21. (3) $\begin{array}{r} 943 \\ -\ \ X \\ \hline 274 \end{array}$

22. (3) $\begin{array}{r} 605 \\ +\ \ M \\ \hline 927 \end{array}$

23. (3) $\begin{array}{r} K \\ -\ 2257 \\ \hline 925 \end{array}$

24. (4) $25 \times N = 400$

25. (4) $625 \div W = 25$

26. (4) $\dfrac{X}{54} = 7$

Add:

27. (1) $\begin{array}{r} 408,627 \\ 915,634 \\ 589,062 \\ +\ 113,093 \\ \hline \end{array}$

28. (1) $\begin{array}{r} 957,125 \\ 826,015 \\ 902,121 \\ +\ 313,947 \\ \hline \end{array}$

29. (1) $73 + 816 + 92 + 47 + 321 + 5432$

30. (1) 92¢ + \$31.82 + \$21

LESSON 5 Addition and Subtraction Word Problems

The key to working word problems that have an addition pattern or a subtraction pattern is recognizing that the problem has a particular pattern.

ADDITION PATTERN	SUBTRACTION PATTERN
$\begin{array}{r} 4 \\ +\ 5 \\ \hline 9 \end{array}$ (9 ← largest number)	$\begin{array}{r} 9 \\ -\ 4 \\ \hline 5 \end{array}$ (9 ← largest number)

Both patterns have three numbers. Word problems that have one of these patterns will give us two of the numbers. We put the given numbers in the pattern and add or subtract as required to find the third number in the pattern.

If a problem makes us think **some and some more,** the pattern is an addition pattern. If a problem makes us think **some went away,** the pattern is a subtraction pattern. The words **how much greater** or **how much less** also indicate a subtraction pattern.

example 5.1 The Duke counted 5260 people. Then the Duchess counted 4720 people. How much greater was the Duke's count?

solution The words *how much greater* tell us that the pattern is a subtraction pattern. We remember that the bottom number in a subtraction pattern is the difference. The greatest number is the minuend (top number).

$$\begin{array}{rl} 5260 & \text{Duke} \\ -\ 4720 & \text{Duchess} \\ \hline 540 & \text{difference} \end{array}$$

The Duke's count was **540** greater than the Duchess's count.

example 5.2 Four hundred seventeen birds were in the trees at eight o'clock. By nine o'clock the number had increased to nine hundred forty-two. How many birds arrived between eight and nine o'clock?

solution This problem makes us think *some and some more*. This means that the problem has an addition pattern. We know one addend and the sum.

$$\begin{array}{rl} 417 & \text{at eight o'clock} \\ +\ \ B & \text{birds arrived} \\ \hline 942 & \text{at nine o'clock} \end{array}$$

This pattern is an addition pattern, but we must subtract the known addend from the sum to find the missing number.

$$\begin{array}{r} 942 \\ -\ 417 \\ \hline 525 \end{array} \quad \xrightarrow{\text{check}} \quad \begin{array}{rl} 417 & \text{at eight o'clock} \\ +\ 525 & \text{birds arrived} \\ \hline 942 & \text{at nine o'clock} \end{array}$$

We find that **525 birds** arrived between eight and nine o'clock.

example 5.3 At sunrise many ducks were on the pond. Then four hundred twenty-five ducks flew away. Six hundred forty-two ducks remained on the pond. How many ducks were on the pond at sunrise?

solution A problem with the premise *some went away* has a subtraction pattern. This problem says some ducks flew away, so the pattern is a subtraction pattern.

D	ducks on the pond at sunrise
− 425	flew away
642	ducks remaining

This is an easy pattern to solve because the minuend (top number) is the sum of the other two numbers.

	check		
425	→	1067	ducks on the pond at sunrise
+ 642		− 425	flew away
1067		642	ducks remaining

There were **1067 ducks** on the pond at sunrise.

example 5.4 George had five dollars and forty-two cents. Then his mom gave him some money. Now he has nine dollars and sixty-five cents. How much money did his mom give him?

solution This problem makes us think *some and some more*. This means the pattern is an addition pattern.

\$5.42	George had
+ M	money his mom gave him
\$9.65	total

This is an addition pattern, but we must subtract to find the missing number. Then we check.

	check		
\$9.65	→	\$5.42	George had
− \$5.42		+ \$4.23	money his mom gave him
\$4.23		\$9.65	total

George's mom gave him **\$4.23.**

practice

a. Hundreds of knights came to the tournament. Then four hundred twenty knights went home. Seven hundred fifty-six knights remained. How many knights came to the tournament?

b. Rita had nine thousand, thirty-five items in her basement. Then she found some more items. Now she has ten thousand, eighty-one items. How many items did she find?

problem set 5

1. (5) When the major estimated the crowd, she estimated 6190 people. Then the captain estimated the crowd and got 5320 people. How much greater was the major's estimate?

2. (5) At nine o'clock there were five hundred thirty dancers in the meadow. By eleven o'clock the number had increased to seven hundred seventy-eight. How many dancers came to the meadow between nine and eleven o'clock?

3. (5) The estimated world population in the year 1650 was 550,000,000. The estimated world population in 1750 was 725,000,000. By how many people did the world's population increase between 1650 and 1750?

4. (5) Zechariah brought \$25.17 with him to the fifth annual carnival of finger painting. He left with \$8.56. How much money did he spend at the carnival?

5. (1) A number has eight digits. All of them are 8 except the ten-thousands' digit, which is 3, the units' digit, which is 7, and the millions' digit, which is 6. What is the number?

6. (1) A number has seven digits. All the digits are 5 except for the hundreds' digit and the millions' digit, both of which are 1. What is the number?

Divide:

7. (4) $\frac{9300}{7}$

8. (4) \$41.63 ÷ 23

9. (4) $16\overline{)41{,}725}$

Multiply:

10. (4) $\begin{array}{r} \$7.89 \\ \times \quad 9 \\ \hline \end{array}$

11. (4) $\begin{array}{r} 506 \\ \times \ 27 \\ \hline \end{array}$

12. (4) $\begin{array}{r} 512 \\ \times \ 632 \\ \hline \end{array}$

Find the missing numbers:

13. (3) $\begin{array}{r} A \\ -\ 526 \\ \hline 437 \end{array}$

14. (3) $\begin{array}{r} 743 \\ -\quad N \\ \hline 188 \end{array}$

15. (3) $F - 123 = 289$

16. (3) $\begin{array}{r} K \\ +\ 325 \\ \hline 1001 \end{array}$

17. (3) $\begin{array}{r} 643 \\ +\quad X \\ \hline 1112 \end{array}$

18. (3) $S + 727 = 915$

19. (4) $\begin{array}{r} 32 \\ \times\ W \\ \hline 224 \end{array}$

20. (4) $D \div 26 = 7$

21. (4) $\frac{990}{P} = 45$

22. (1) Write in standard notation: $(7 \times 10{,}000) + (4 \times 100) + (3 \times 10)$

23. (1) Write 2109 in expanded notation.

24. (1) Use digits to write forty-seven million, fourteen.

25. (1) Use digits to write fourteen billion, forty-two thousand, seven hundred fifty-five.

26. (2) Write these numbers in order from least to greatest: 916, 169, –619, 961, 196, –691

27. (1) Use words to write 5021.

28. (2) Round 716,487,250 to the nearest thousand.

Add:

29. (1) 408,627 + 915,634

30. (1) \$9.99 + 33¢ + \$1.07

LESSON 6 *Reading and Writing Decimal Numbers • Adding and Subtracting Decimal Numbers • Rounding Decimal Numbers*

6.A reading and writing decimal numbers

We have noted that whole numbers have a decimal point just after the last digit in the number. Sometimes the decimal point is written. Many times it is not written but is understood to be there. Thus, the two notations

615. and 615

both designate the number six hundred fifteen.

Some numbers have digits to the right of the decimal point. We often call these numbers **decimal numbers.** Some people call these numbers **decimal fractions** because they can be written as whole numbers divided by 10, 100, 1000, 10,000, or some other multiple of 10, as shown here.

$$61.23 = \frac{6123}{100}$$

The first place to the right of the decimal in a decimal fraction is the tenths' place, which has a place value of $\frac{1}{10}$. The next place is the hundredths' place, with a place value of $\frac{1}{100}$, followed by the thousandths' place, with a place value of $\frac{1}{1000}$, and so on. The value of a digit is the digit times the place value. Thus, the 6 in

0.0006724

has a value of 6 times $\frac{1}{10,000}$, or 6 ten-thousandths, because it is in the ten-thousandths' place. When a whole number does not precede the decimal number, a zero is placed before the decimal point.

Decimal Place Values

1,000,000	millions
,	
100,000	hundred thousands
10,000	ten thousands
1,000	thousands
,	
100	hundreds
10	tens
1	units
.	decimal point
$\frac{1}{10}$	tenths
$\frac{1}{100}$	hundredths
$\frac{1}{1,000}$	thousandths
$\frac{1}{10,000}$	ten-thousandths
$\frac{1}{100,000}$	hundred-thousandths
$\frac{1}{1,000,000}$	millionths

To read a decimal number, we begin on the left-hand end and read according to the following procedure:

1. Read the digits to the left of the decimal point in the same way you would read a whole number.
2. Read the decimal point as *and*.
3. Read the digits to the right of the decimal point as if they formed a whole number; then name the *place value* of the last digit in the number.

When writing out the decimal part of a number with words, place the commas in the same places you would when writing out whole numbers. Note, however, that commas are *not* used in the numerical representation of decimals.

We will demonstrate by reading several decimal fractions.

NUMBER	READ AS
0.413	four hundred thirteen thousandths
0.041301	forty-one thousand, three hundred one millionths
0.70265	seventy thousand, two hundred sixty-five hundred-thousandths
0.0412	four hundred twelve ten-thousandths

In the following example we will read two decimal numbers that have nonzero digits on both sides of the decimal point.

example 6.1 Read the numbers, then write them with words: (a) 4165.0162 (b) 7108000.21578

solution (a) 4165.0162 **Four thousand, one hundred sixty-five *and* one hundred sixty-two ten-thousandths**

(b) 7,108,000.21578 **Seven million, one hundred eight thousand *and* twenty-one thousand, five hundred seventy-eight hundred-thousandths**

To write a decimal number using digits and a decimal point:

1. Write the whole number part as usual.
2. Represent *and* with a decimal point.
3. Find the place value of the first digit by looking at the final word.
4. Fill in the decimal part of the number by writing that part after *and* so that its final digit has the correct place value.
5. If necessary, fill in any blank places between the decimal point and the number just written with zeros.

example 6.2 Use digits and a decimal point to write each number:

(a) Fifteen and thirty-one hundredths

(b) Two hundred three and twenty-five thousandths

solution

(a) Fifteen	15
and	15.
… hundredths	15.__
thirty-one	15.3 1
	15.31
(b) Two hundred three	203
and	203.
… thousandths	203.___
twenty-five	203._ 2 5
	203.025

6.B adding and subtracting decimal numbers

We add and subtract decimal numbers just as we do whole numbers. When we add and subtract decimal numbers, we must remember to write the numbers so that the decimal points are aligned one above the other, just as we vertically align the digits in the units' place in whole number addition.

example 6.3 Simplify: (a) 6.231 + 0.04 (b) 6.23 − 0.044

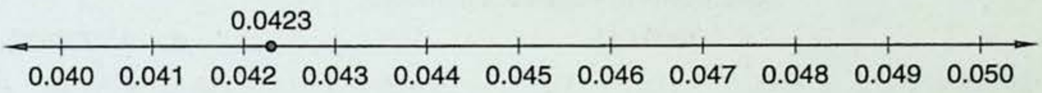
solution To begin, we write the numbers with the decimal points one above the other. In these examples, we write place-holding zeros to the right of a decimal number to make the arithmetic easier. Then we add or subtract as indicated.

(a)
$$\begin{array}{r} 6.231 \\ +\ 0.040 \\ \hline \mathbf{6.271} \end{array}$$

(b)
$$\begin{array}{r} 6.2{}^{1}{}^{1}\!\not{2}\,{}^{2}\!\not{3}\,{}^{1}0 \\ -\ 0.044 \\ \hline \mathbf{6.186} \end{array}$$

6.C rounding decimal numbers

The number line can help us understand the process of rounding decimal numbers. If we graph 0.0423,



we can see that this number is closer to 0.042 than it is to 0.043. If we round 0.0423 to the nearest thousandth, we get

0.042

Since 0.0423 is closer to 0.04 than it is to 0.05, we can also round 0.0423 to the nearest hundredth, or

0.04

We find that a circle and an arrow can also be used to explain how we round decimal numbers. The procedure is the same as the procedure we use for whole numbers.

example 6.4 Round 212.0165725 to the nearest ten-thousandth.

solution First we circle the digit in the ten-thousandths' place. Then we mark the digit to its right with an arrow.

↓
212.016⑤725

The arrow-marked digit is greater than 5, so we increase the circled digit by 1.

↓
212.016⑥725

Next we change the arrow-marked digit and the digits to its right to zero.

↓
212.016⑥000

Zeros at the end of the number and to the right of the decimal point have no value, so we can omit them.

212.0166

example 6.5 Round 4057.2138362 to two decimal places.

solution We begin by circling the second digit to the right of the decimal point and marking the next digit to its right with an arrow.

↓
4057.2①38362

Since the arrow-marked digit is less than 5, we do not change the circled digit. Now we change the arrow-marked digit and the digits to its right to 0. We discard the terminal zeros and get

4057.21

When using a calculator, it is common to have an answer result in many decimal places. It is important to know how to accurately round decimal numbers.

practice Use digits and a decimal point to write each number:

a. Five thousand and seven hundred forty-two ten-thousandths

b. Six hundred forty-two and seventy-five thousandths

Use words to write each number:

c. 7000.065 **d.** 42.000617

Add or subtract as indicated:

e. 44.016 − 0.1422 **f.** 5.228 + 6.759

g. Round 416.042737 to the nearest hundred-thousandth.

h. Round 2837.065248 to the nearest ten-thousandth.

problem set 6

1. *(5)* Thirty-seven million, nine hundred eighteen thousand, five hundred is how much greater than nineteen million, ninety-nine thousand, nine?

2. *(5)* The estimated population of the world in 1750 was 725,000,000 people. During the next 100 years the population increased by 475,000,000 people. What was the world's population in 1850?

3. *(5)* Exactly 10,000 runners began the marathon. If only 5420 runners finished the marathon, how many dropped out along the way?

4. *(1)* A number has seven digits. All the digits are 6 except the hundred-thousands' digit, which is 2, and the thousands' digit, which is 4. What is the number?

5. *(1)* A number has six digits. All the digits are 3 except the ten-thousands' digit, which is 7, and the thousands' digit, which is 2. What is the number?

Subtract. Add to check.

6. *(6)* 14.03 − 0.0132 **7.** *(6)* 941.2 − 14.23

Divide:

8. *(4)* $\frac{3624}{23}$ **9.** *(4)* \$12.75 ÷ 17 **10.** *(4)* $51\overline{)41,362}$

11. *(4)* $27\overline{)2189}$ **12.** *(4)* 2546 ÷ 41 **13.** *(4)* $\frac{92,438}{51}$

14. *(6)* Round 0.020532 to the nearest thousandth.

15. *(6)* Round 14.11931627 to the nearest hundred-thousandth.

Use words to write each number:

16. *(6)* 3178.0285 **17.** *(6)* 504327.001510512

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