Student Solutions Manual

CALCULUS

EARLY TRANSCENDENTAL FUNCTIONS

with CalcChat® and CalcView®

7e

RON LARSON
BRUCE EDWARDS

RON LARSON | BRUCE EDWARDS

CONTENTS

CHAPTER 1
Preparation for Calculus

Section 1.1 Graphs and Models

1. To find the x-intercepts of the graph of an equation, let y be zero and solve the equation for x. To find the y-intercepts of the graph of an equation, let x be zero and solve the equation for y.

3. $y = -\frac{3}{2}x + 3$

x-intercept: $(2, 0)$

y-intercept: $(0, 3)$

Matches graph (b).

4. $y = \sqrt{9 - x^2}$

x-intercepts: $(-3, 0), (3, 0)$

y-intercept: $(0, 3)$

Matches graph (d).

5. $y = 3 - x^2$

x-intercepts: $\left(\sqrt{3}, 0\right), \left(-\sqrt{3}, 0\right)$

y-intercept: $(0, 3)$

Matches graph (a).

6. $y = x^3 - x$

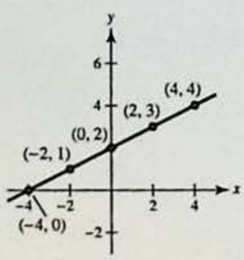
x-intercepts: $(0, 0), (-1, 0), (1, 0)$

y-intercept: $(0, 0)$

Matches graph (c).

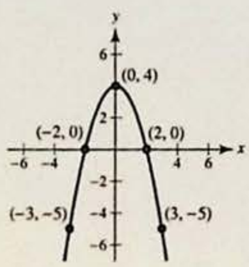
7. $y = \frac{1}{2}x + 2$

x	-4	-2	0	2	4
y	0	1	2	3	4



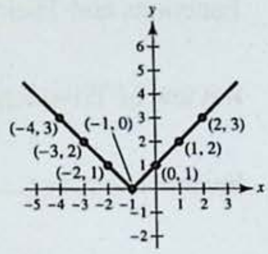
9. $y = 4 - x^2$

x	-3	-2	0	2	3
y	-5	0	4	0	-5



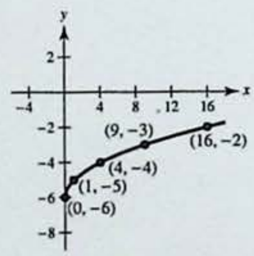
11. $y = |x + 1|$

x	-4	-3	-2	-1	0	1	2
y	3	2	1	0	1	2	3



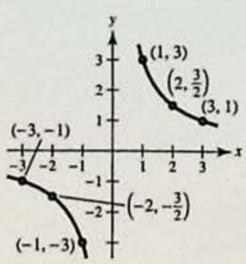
13. $y = \sqrt{x} - 6$

x	0	1	4	9	16
y	-6	-5	-4	-3	-2

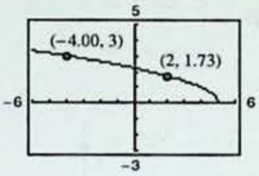


15. $y = \dfrac{3}{x}$

x	-3	-2	-1	0	1	2	3
y	-1	$-\frac{3}{2}$	-3	Undef.	3	$\frac{3}{2}$	1



17. $y = \sqrt{5 - x}$



(a) $(2, y) = (2, 1.73) \quad \left(y = \sqrt{5 - 2} = \sqrt{3} \approx 1.73\right)$

(b) $(x, 3) = (-4, 3) \quad \left(3 = \sqrt{5 - (-4)}\right)$

19. $y = 2x - 5$

y-intercept: $y = 2(0) - 5 = -5;\ (0, -5)$

x-intercept: $0 = 2x - 5$

$5 = 2x$

$x = \frac{5}{2};\ \left(\frac{5}{2}, 0\right)$

21. $y = x^2 + x - 2$

y-intercept: $y = 0^2 + 0 - 2$

$y = -2;\ (0, -2)$

x-intercepts: $0 = x^2 + x - 2$

$0 = (x + 2)(x - 1)$

$x = -2, 1;\ (-2, 0), (1, 0)$

23. $y = x\sqrt{16 - x^2}$

y-intercept: $y = 0\sqrt{16 - 0^2} = 0;\ (0, 0)$

x-intercepts: $0 = x\sqrt{16 - x^2}$

$0 = x\sqrt{(4 - x)(4 + x)}$

$x = 0, 4, -4;\ (0, 0), (4, 0), (-4, 0)$

25. $y = \dfrac{2 - \sqrt{x}}{5x + 1}$

y-intercept: $y = \dfrac{2 - \sqrt{0}}{5(0) + 1} = 2\ ;\ (0, 2)$

x-intercept: $0 = \dfrac{2 - \sqrt{x}}{5x + 1}$

$0 = 2 - \sqrt{x}$

$x = 4\ ;\ (4, 0)$

27. $x^2y - x^2 + 4y = 0$

y-intercept: $0^2(y) - 0^2 + 4y = 0$

$y = 0;\ (0, 0)$

x-intercept: $x^2(0) - x^2 + 4(0) = 0$

$x = 0;\ (0, 0)$

29. Symmetric with respect to the y-axis because $y = (-x)^2 - 6 = x^2 - 6$.

31. Symmetric with respect to the x-axis because $(-y)^2 = y^2 = x^3 - 8x$.

33. Symmetric with respect to the origin because $(-x)(-y) = xy = 4$.

35. $y = 4 - \sqrt{x + 3}$

No symmetry with respect to either axis or the origin.

37. Symmetric with respect to the origin because

$$-y = \frac{-x}{(-x)^2 + 1}$$

$$y = \frac{x}{x^2 + 1}.$$

39. $y = \left|x^3 + x\right|$ is symmetric with respect to the y-axis because $y = \left|(-x)^3 + (-x)\right| = \left|-(x^3 + x)\right| = \left|x^3 + x\right|$.

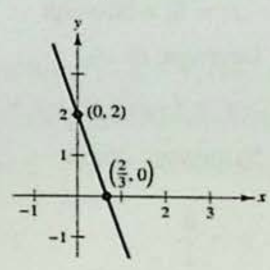
41. $y = 2 - 3x$

$y = 2 - 3(0) = 2$, y-intercept

$0 = 2 - 3(x) \Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3}$, x-intercept

Intercepts: $(0, 2), \left(\frac{2}{3}, 0\right)$

Symmetry: none



43. $y = 9 - x^2$

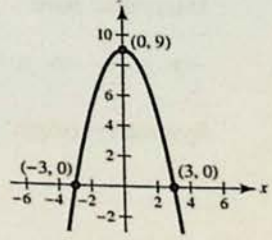
$y = 9 - (0)^2 = 9$, y-intercept

$0 = 9 - x^2 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$, x-intercepts

Intercepts: $(0, 9), (3, 0), (-3, 0)$

$y = 9 - (-x)^2 = 9 - x^2$

Symmetry: y-axis



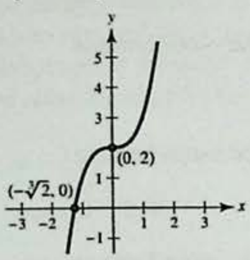
45. $y = x^3 + 2$

$y = 0^3 + 2 = 2$, y-intercept

$0 = x^3 + 2 \Rightarrow x^3 = -2 \Rightarrow x = -\sqrt[3]{2}$, x-intercept

Intercepts: $\left(-\sqrt[3]{2}, 0\right)$, $(0, 2)$

Symmetry: none



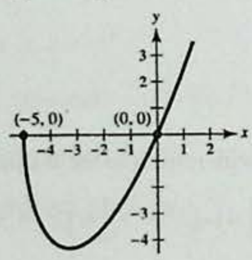
47. $y = x\sqrt{x + 5}$

$y = 0\sqrt{0 + 5} = 0$, y-intercept

$x\sqrt{x + 5} = 0 \Rightarrow x = 0, -5$, x-intercepts

Intercepts: $(0, 0)$, $(-5, 0)$

Symmetry: none



49. $x = y^3$

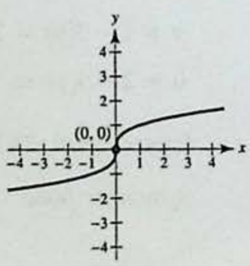
$y^3 = 0 \Rightarrow y = 0$, y-intercept

$x = 0$, x-intercept

Intercept: $(0, 0)$

$-x = (-y)^3 \Rightarrow -x = -y^3$

Symmetry: origin



51. $y = \dfrac{8}{x}$

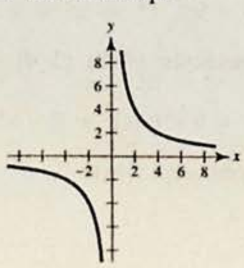
$y = \dfrac{8}{0} \Rightarrow$ Undefined $\Rightarrow$ no y-intercept

$\dfrac{8}{x} = 0 \Rightarrow$ No solution $\Rightarrow$ no x-intercept

Intercepts: none

$-y = \dfrac{8}{-x} \Rightarrow y = \dfrac{8}{x}$

Symmetry: origin



53. $y = 6 - |x|$

$y = 6 - |0| = 6$, y-intercept

$6 - |x| = 0$

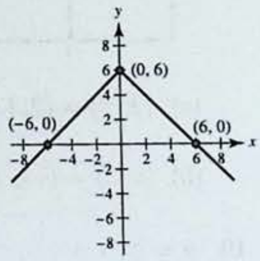
$6 = |x|$

$x = \pm 6$, x-intercepts

Intercepts: $(0, 6)$, $(-6, 0)$, $(6, 0)$

$y = 6 - |-x| = 6 - |x|$

Symmetry: y-axis



55. $3y^2 - x = 9$

$3y^2 = x + 9$

$y^2 = \frac{1}{3}x + 3$

$y = \pm\sqrt{\frac{1}{3}x + 3}$

$y = \pm\sqrt{0 + 3} = \pm\sqrt{3}$, y-intercepts

$\pm\sqrt{\frac{1}{3}x + 3} = 0$

$\frac{1}{3}x + 3 = 0$

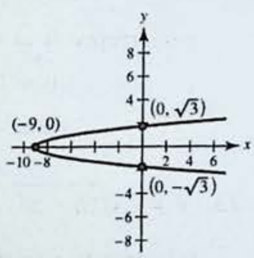
$x = -9$, x-intercept

Intercepts:

$\left(0, \sqrt{3}\right)$, $\left(0, -\sqrt{3}\right)$, $(-9, 0)$

$3(-y)^2 - x = 3y^2 - x = 9$

Symmetry: x-axis



57. $x + y = 8 \Rightarrow y = 8 - x$

$4x - y = 7 \Rightarrow y = 4x - 7$

$8 - x = 4x - 7$

$15 = 5x$

$3 = x$

The corresponding y-value is $y = 5$.

Point of intersection: $(3, 5)$

59. $x^2 + y = 15 \Rightarrow y = -x^2 + 15$

$-3x + y = 11 \Rightarrow y = 3x + 11$

$-x^2 + 15 = 3x + 11$

$0 = x^2 + 3x - 4$

$0 = (x + 4)(x - 1)$

$x = -4, 1$

The corresponding y-values are $y = -1$ (for $x = -4$) and $y = 14$ (for $x = 1$).

Points of intersection: $(-4, -1)$, $(1, 14)$

61. $x^2 + y^2 = 5 \Rightarrow y^2 = 5 - x^2$

$x - y = 1 \Rightarrow y = x - 1$

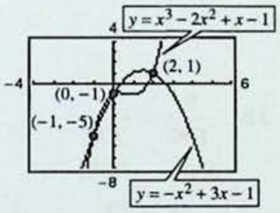
$$5 - x^2 = (x - 1)^2$$
$$5 - x^2 = x^2 - 2x + 1$$
$$0 = 2x^2 - 2x - 4 = 2(x + 1)(x - 2)$$
$$x = -1 \text{ or } x = 2$$

The corresponding y-values are $y = -2$ (for $x = -1$) and $y = 1$ (for $x = 2$).

Points of intersection: $(-1, -2)$, $(2, 1)$

63. $y = x^3 - 2x^2 + x - 1$

$y = -x^2 + 3x - 1$



Points of intersection:

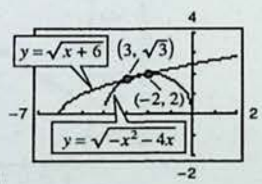
$(-1, -5)$, $(0, -1)$, $(2, 1)$

Analytically,

$$x^3 - 2x^2 + x - 1 = -x^2 + 3x - 1$$
$$x^3 - x^2 - 2x = 0$$
$$x(x - 2)(x + 1) = 0$$
$$x = -1, 0, 2.$$

65. $y = \sqrt{x + 6}$

$y = \sqrt{-x^2 - 4x}$



Points of intersection:

$(-2, 2)$, $\left(-3, \sqrt{3}\right) \approx (-3, 1.732)$

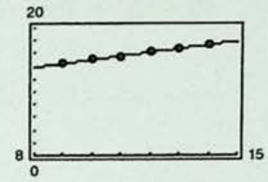
Analytically,

$$\sqrt{x + 6} = \sqrt{-x^2 - 4x}$$
$$x + 6 = -x^2 - 4x$$
$$x^2 + 5x + 6 = 0$$
$$(x + 3)(x + 2) = 0$$
$$x = -3, -2.$$

67. (a) Using a graphing utility, you obtain

$y = 0.58t + 9.2.$

(b)



The model is a good fit for the data.

(c) For 2024, $t = 24$:

$y = 0.58(24) + 9.2 \approx 23.1$

The GDP in 2024 will be approximately $23.1 trillion.

69.
$$C = R$$
$$2.04x + 5600 = 3.29x$$
$$5600 = 3.29x - 2.04x$$
$$5600 = 1.25x$$
$$x = \frac{5600}{1.25} = 4480$$

To break even, 4480 units must be sold.

71. Answers may vary. *Sample answer*:

$y = \left(x + \frac{3}{2}\right)(x - 4)\left(x - \frac{5}{2}\right)$ has intercepts at $x = -\frac{3}{2}$, $x = 4$, and $x = \frac{5}{2}$.

73. Yes. Assume that the graph has x-axis and origin symmetry. If (x, y) is on the graph, so is $(x, -y)$ by x-axis symmetry. Because $(x, -y)$ is on the graph, then so is $(-x, -(-y)) = (-x, y)$ by origin symmetry. Therefore, the graph is symmetric with respect to the y-axis. The argument is similar for y-axis and origin symmetry.

75. False. x-axis symmetry means that if $(-4, -5)$ is on the graph, then $(-4, 5)$ is also on the graph. For example, $(4, -5)$ is not on the graph of $x = y^2 - 29$, whereas $(-4, -5)$ is on the graph.

77. True. The x-intercepts are $\left(\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}, 0\right)$.

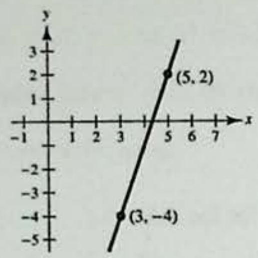
Section 1.2 Linear Models and Rates of Change

1. In the form $y = mx + b$, m is the slope and b is the y-intercept.

3. $m = 2$

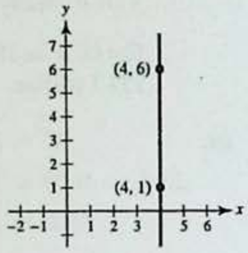
5. $m = -1$

7. $m = \dfrac{2 - (-4)}{5 - 3} = \dfrac{6}{2} = 3$

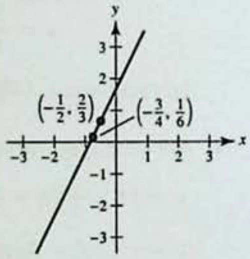


9. $m = \dfrac{1 - 6}{4 - 4} = \dfrac{-5}{0}$, undefined.

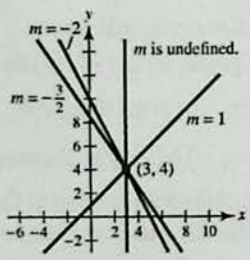
The line is vertical.



11. $m = \dfrac{\frac{2}{3} - \frac{1}{6}}{-\frac{1}{2} - \left(-\frac{3}{4}\right)} = \dfrac{\frac{1}{2}}{\frac{1}{4}} = 2$



13.



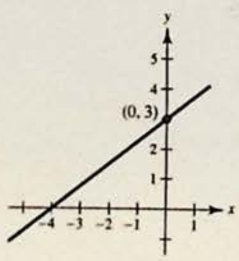
15. Because the slope is 0, the line is horizontal and its equation is $y = 2$. Therefore, three additional points are $(0, 2)$, $(1, 2)$, $(5, 2)$.

17. The equation of this line is

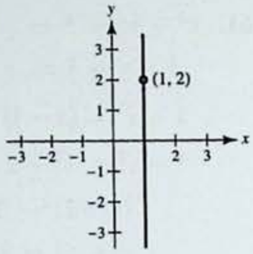
$$y - 7 = -3(x - 1)$$
$$y = -3x + 10.$$

Therefore, three additional points are $(0, 10)$, $(2, 4)$, and $(3, 1)$.

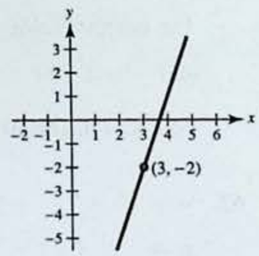
19.
$$y = \tfrac{3}{4}x + 3$$
$$4y = 3x + 12$$
$$0 = 3x - 4y + 12$$



21. Because the slope is undefined, the line is vertical and its equation is $x = 1$.



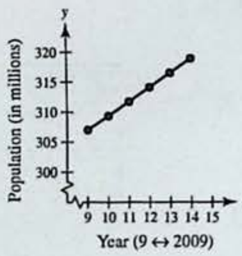
23.
$$y + 2 = 3(x - 3)$$
$$y + 2 = 3x - 9$$
$$y = 3x - 11$$
$$0 = 3x - y - 11$$



25.
$$\frac{6}{100} = \frac{x}{200}$$
$$100x = 1200$$
$$x = 12$$

Since the grade of the road is $\dfrac{6}{100}$, if you drive 200 feet, the vertical rise in the road will be 12 feet.

27. (a)



Slopes: $\dfrac{309.3 - 307.0}{10 - 9} = 2.3$

$\dfrac{311.7 - 309.3}{11 - 10} = 2.4$

$\dfrac{314.1 - 311.7}{12 - 11} = 2.4$

$\dfrac{316.5 - 314.1}{13 - 12} = 2.4$

$\dfrac{318.9 - 316.5}{14 - 13} = 2.4$

The population increased least rapidly from 2009 to 2010.

(b) $\dfrac{318.9 - 307.0}{14 - 9} = 2.38$ million people per year

(c) For 2025, $t = 25$:

$$\frac{P - 307.0}{25 - 9} = 2.38 \Rightarrow P = 2.38(16) + 307.0$$
$$\approx 345.1$$

The population of the United States in 2025 will be about 345.1 million people.

29. $y = 4x - 3$

The slope is $m = 4$ and the y-intercept is $(0, -3)$.

31. $5x + y = 20$

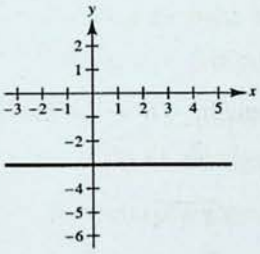
$$y = -5x + 20$$

The slope is $m = -5$ and the y-intercept is $(0, 20)$.

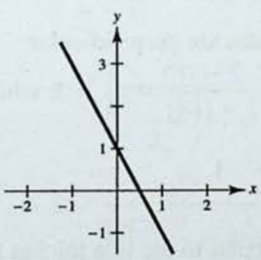
33. $x = 4$

The line is vertical. Therefore, the slope is undefined and there is no y-intercept.

35. $y = -3$

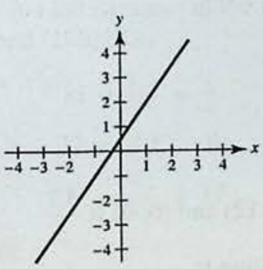


37. $y = -2x + 1$



39. $y - 2 = \frac{3}{2}(x - 1)$

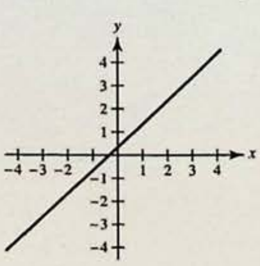
$$y = \frac{3}{2}x + \frac{1}{2}$$



41. $3x - 3y + 1 = 0$

$$3y = 3x + 1$$

$$y = x + \frac{1}{3}$$

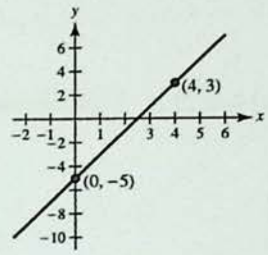


43. $m = \dfrac{-5 - 3}{0 - 4} = \dfrac{-8}{-4} = 2$

$$y - (-5) = 2(x - 0)$$

$$y + 5 = 2x$$

$$0 = 2x - y - 5$$

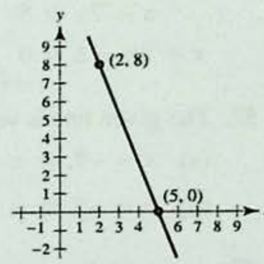


45. $m = \dfrac{8 - 0}{2 - 5} = -\dfrac{8}{3}$

$$y - 0 = -\frac{8}{3}(x - 5)$$

$$y = -\frac{8}{3}x + \frac{40}{3}$$

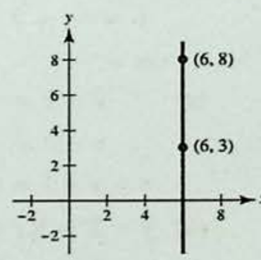
$$8x + 3y - 40 = 0$$



47. $m = \dfrac{8 - 3}{6 - 6} = \dfrac{5}{0}$, undefined

The line is vertical.

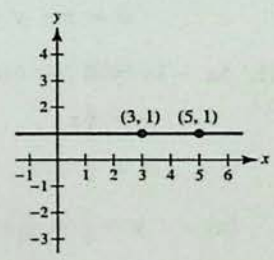
$x = 6$ or $x - 6 = 0$



49. $m = \dfrac{1 - 1}{5 - 3} = 0$

The line is horizontal.

$y = 1$ or $y - 1 = 0$



51. The slope is $\dfrac{1 - b}{3 - 0} = \dfrac{1 - b}{3}$.

The y-intercept is $(0, b)$. Hence,

$$y = mx + b = \left(\frac{1 - b}{3}\right)x + b.$$

53. $\dfrac{x}{2} + \dfrac{y}{3} = 1$

$$3x + 2y - 6 = 0$$

55. $\frac{x}{2a} + \frac{y}{a} = 1$

$\frac{9}{2a} + \frac{-2}{a} = 1$

$\frac{9-4}{2a} = 1$

$5 = 2a$

$a = \frac{5}{2}$

$\frac{x}{2\left(\frac{5}{2}\right)} + \frac{y}{\left(\frac{5}{2}\right)} = 1$

$\frac{x}{5} + \frac{2y}{5} = 1$

$x + 2y = 5$

$x + 2y - 5 = 0$

57. The given line is vertical.

(a) $x = -7$, or $x + 7 = 0$

(b) $y = -2$, or $y + 2 = 0$

59. $x + y = 7$

$y = -x + 7$

$m = -1$

(a) $y - 2 = -1(x + 3)$

$y - 2 = -x - 3$

$x + y + 1 = 0$

(b) $y - 2 = 1(x + 3)$

$y - 2 = x + 3$

$0 = x - y + 5$

61. $5x - 3y = 0$

$y = \frac{5}{3}x$

$m = \frac{5}{3}$

(a) $y - \frac{7}{8} = \frac{5}{3}\left(x - \frac{3}{4}\right)$

$24y - 21 = 40x - 30$

$0 = 40x - 24y - 9$

(b) $y - \frac{7}{8} = -\frac{3}{5}\left(x - \frac{3}{4}\right)$

$40y - 35 = -24x + 18$

$24x + 40y - 53 = 0$

63. The slope is 250.

$V = 1850$ when $t = 6$.

$V = 250(t - 6) + 1850$

$= 250t + 250$

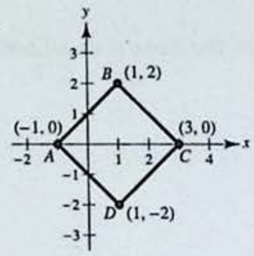
65. $m_1 = \frac{1-0}{-2-(-1)} = -1$

$m_2 = \frac{-2-0}{2-(-1)} = -\frac{2}{3}$

$m_1 \neq m_2$

The points are not collinear.

67.



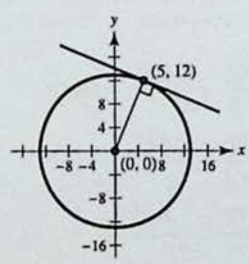
The four sides are of equal length: $\sqrt{8} = 2\sqrt{2}$.

For example, the length of segment AB is

$$\sqrt{(1-(-1))^2 + (2-0)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \text{ units.}$$

Furthermore, the adjacent sides are perpendicular because the slope of $\overline{AB}$ is $\frac{2-0}{1-(-1)} = \frac{2}{2} = 1$, whereas the slope of $\overline{BC}$ is $\frac{2-0}{1-3} = -1$.

69. The tangent line is perpendicular to the line joining the point (5, 12) and the center (0, 0).



Slope of the line joining (5, 12) and (0, 0) is $\frac{12}{5}$.

The equation of the tangent line is

$y - 12 = \frac{-5}{12}(x - 5)$

$y = \frac{-5}{12}x + \frac{169}{12}$

$5x + 12y - 169 = 0.$

71. (a) The slope of the segment joining (b, c) and $(a, 0)$ is $\dfrac{c}{(b-a)}$. The slope of the perpendicular bisector of this segment is $\dfrac{(a-b)}{c}$. The midpoint of this segment is $\left(\dfrac{a+b}{2}, \dfrac{c}{2}\right)$.

So, the equation of the perpendicular bisector to this segment is

$$y - \frac{c}{2} = \frac{a-b}{c}\left(x - \frac{a+b}{2}\right).$$

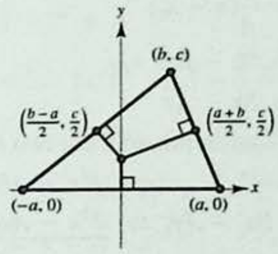
Similarly, the equation of the perpendicular bisector of the segment joining $(-a, 0)$ and $(a, 0)$ is

$$y - \frac{c}{2} = \frac{a-b}{-c}\left(x - \frac{b-a}{2}\right).$$

Equating the right-hand sides of each equation, you obtain $x = 0$.

Letting $x = 0$ in either equation yields the point of intersection:

$$y = \frac{c}{2} + \frac{a-b}{c}\left(0 - \frac{a+b}{2}\right) = \frac{c^2}{2c} + \frac{b^2 - a^2}{2c} = \frac{c^2 + b^2 - a^2}{2c}.$$



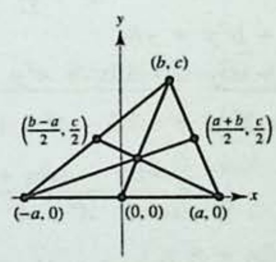
The point of intersection is $\left(0, \dfrac{-a^2 + b^2 + c^2}{2c}\right)$.

(b) The equations of the medians are:

$$y = \frac{c}{b}x$$

$$y = \frac{c/2}{\left(\dfrac{b-a}{2}\right) - a}(x - a) = \frac{c}{b - 3a}(x - a)$$

$$y = \frac{c/2}{\left(\dfrac{a+b}{2} + a\right)}(x + a) = \frac{c}{3a + b}(x + a).$$



Solving these equation simultaneously for (x, y), you obtain the point of intersection $\left(\dfrac{b}{3}, \dfrac{c}{3}\right)$.

73. Find the equation of the line through the points (0, 32) and (100, 212).

$$m = \tfrac{180}{100} = \tfrac{9}{5}$$

$$F - 32 = \tfrac{9}{5}(C - 0)$$

$$F = \tfrac{9}{5}C + 32$$

or

$$C = \tfrac{1}{9}(5F - 160)$$

$$5F - 9C - 160 = 0$$

For $F = 72°$, $C \approx 22.2°$.

75. (a) Two points are (50, 780) and (47, 825). The slope is

$$m = \frac{825 - 780}{47 - 50} = \frac{45}{-3} = -15.$$

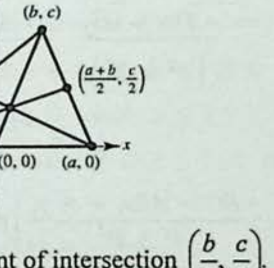
$$p - 780 = -15(x - 50)$$

$$p = -15x + 750 + 780 = -15x + 1530$$

or

$$x = \frac{1}{15}(1530 - p)$$

(b)



If $p = 855$, then $x = 45$ units.

(c) If $p = 795$, then $x = \dfrac{1}{15}(1530 - 795) = 49$ units

77. If $A = 0$, then $By + C = 0$ is the horizontal line $y = -C/B$. The distance to (x_1, y_1) is

$$d = \left|y_1 - \left(\frac{-C}{B}\right)\right| = \frac{|By_1 + C|}{|B|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

If $B = 0$, then $Ax + C = 0$ is the vertical line $x = -C/A$. The distance to (x_1, y_1) is

$$d = \left|x_1 - \left(\frac{-C}{A}\right)\right| = \frac{|Ax_1 + C|}{|A|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

(Note that A and B cannot both be zero.) The slope of the line $Ax + By + C = 0$ is $-A/B$.

The equation of the line through (x_1, y_1) perpendicular to $Ax + By + C = 0$ is:

$$y - y_1 = \frac{B}{A}(x - x_1)$$
$$Ay - Ay_1 = Bx - Bx_1$$
$$Bx_1 - Ay_1 = Bx - Ay$$

The point of intersection of these two lines is:

$$Ax + By = -C \quad \Rightarrow A^2x + ABy = -AC \qquad (1)$$
$$Bx - Ay = Bx_1 - Ay_1 \Rightarrow \underline{B^2x - ABy} = \underline{B^2x_1 - ABy_1} \qquad (2)$$
$$(A^2 + B^2)x = -AC + B^2x_1 - ABy_1 \quad \text{(By adding equations (1) and (2))}$$
$$x = \frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2}$$

$$Ax + By = -C \quad \Rightarrow ABx + B^2y = -BC \qquad (3)$$
$$Bx - Ay = Bx_1 - Ay_1 \Rightarrow \underline{-ABx + A^2y} = \underline{-ABx_1 + A^2y_1} \qquad (4)$$
$$(A^2 + B^2)y = -BC - ABx_1 + A^2y_1 \quad \text{(By adding equations (3) and (4))}$$
$$y = \frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2}$$

$$\left(\frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2}, \frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2}\right) \text{ point of intersection}$$

The distance between (x_1, y_1) and this point gives you the distance between (x_1, y_1) and the line $Ax + By + C = 0$.

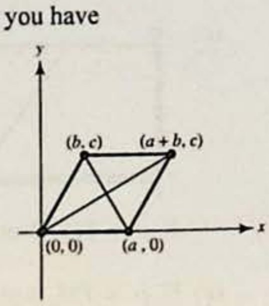
79. $x - y - 2 = 0 \Rightarrow d = \dfrac{|1(-2) + (-1)(1) - 2|}{\sqrt{1^2 + 1^2}} = \dfrac{5}{\sqrt{2}} = \dfrac{5\sqrt{2}}{2}$

81. For simplicity, let the vertices of the rhombus be $(0, 0)$, $(a, 0)$, (b, c), and $(a + b, c)$, as shown in the figure.

The slopes of the diagonals are then $m_1 = \dfrac{c}{a + b}$

and $m_2 = \dfrac{c}{b - a}$. Because the sides of the rhombus are equal, $a^2 = b^2 + c^2$, and you have

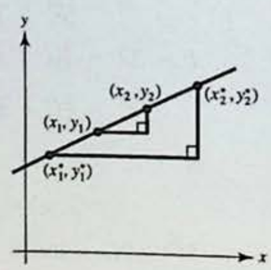
$$m_1m_2 = \frac{c}{a + b} \cdot \frac{c}{b - a}$$
$$= \frac{c^2}{b^2 - a^2}$$
$$= \frac{c^2}{-c^2}$$
$$= -1.$$



Therefore, the diagonals are perpendicular.

83. Consider the figure below in which the four points are collinear. Because the triangles are similar, the result immediately follows.

$$\frac{y_2^* - y_1^*}{x_2^* - x_1^*} = \frac{y_2 - y_1}{x_2 - x_1}$$



85. True.

$$ax + by = c_1 \Rightarrow y = -\frac{a}{b}x + \frac{c_1}{b} \Rightarrow m_1 = -\frac{a}{b}$$

$$bx - ay = c_2 \Rightarrow y = \frac{b}{a}x - \frac{c_2}{a} \Rightarrow m_2 = \frac{b}{a}$$

$$m_2 = -\frac{1}{m_1}$$

Section 1.3 Functions and Their Graphs

1. A relation between two sets X and Y is a set of ordered pairs of the form (x, y), where x is a member of X and y is at member of Y.

A function from X to Y is a relation between X and Y that has the property that any two ordered pairs with the same x-value also have the same y-value.

3. The three basic types are vertical shifts, horizontal shifts, and reflections.

5. $f(x) = 3x - 2$

(a) $f(0) = 3(0) - 2 = -2$

(b) $f(5) = 3(5) - 2 = 13$

(c) $f(b) = 3(b) - 2 = 3b - 2$

(d) $f(x - 1) = 3(x - 1) - 2 = 3x - 5$

7. (a) $f(-2) = \sqrt{(-2)^2 + 4} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$

(b) $f(3) = \sqrt{3^2 + 4} = \sqrt{9 + 4} = \sqrt{13}$

(c) $f(2) = \sqrt{2^2 + 4} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$

(d) $f(x + bx) = \sqrt{(x + bx)^2 + 4}$

$= \sqrt{x^2 + 2bx^2 + b^2x^2 + 4}$

9. (a) $g(0) = 5 - 0^2 = 5$

(b) $g(\sqrt{5}) = 5 - (\sqrt{5})^2 = 5 - 5 = 0$

(c) $g(-2) = 5 - (-2)^2 = 5 - 4 = 1$

(d) $g(t - 1) = 5 - (t - 1)^2 = 5 - (t^2 - 2t + 1)$

$= 4 + 2t - t^2$

11. $$\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{(x + \Delta x)^3 - x^3}{\Delta x} = \frac{x^3 + 3x^2\Delta x + 3x^2(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x} = 3x^2 + 3x\Delta x + (\Delta x)^2,\ \Delta x \neq 0$$

13. $f(x) = 4x^2$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

15. $f(x) = x^3$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

17. $g(x) = \sqrt{6x}$

Domain: $6x \geq 0$

$x \geq 0 \Rightarrow [0, \infty)$

Range: $[0, \infty)$

19. $f(x) = \sqrt{16 - x^2}$

$16 - x^2 \geq 0 \Rightarrow x^2 \leq 16$

Domain: $[-4, 4]$

Range: $[0, 4]$

Note: $y = \sqrt{16 - x^2}$ is a semicircle of radius 4.

21. $f(x) = \dfrac{3}{x}$

Domain: all $x \neq 0 \Rightarrow (-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

23. $f(x) = \sqrt{x} + \sqrt{1 - x}$

$x \geq 0$ and $1 - x \geq 0$

$x \geq 0$ and $x \leq 1$

Domain: $0 \leq x \leq 1 \Rightarrow [0, 1]$

25. $f(x) = \dfrac{1}{|x + 3|}$

$|x + 3| \neq 0$

$x + 3 \neq 0$

Domain: all $x \neq -3$

Domain: $(-\infty, -3) \cup (-3, \infty)$

27. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$

(a) $f(-1) = 2(-1) + 1 = -1$

(b) $f(0) = 2(0) + 2 = 2$

(c) $f(2) = 2(2) + 2 = 6$

(d) $f(t^2 + 1) = 2(t^2 + 1) + 2 = 2t^2 + 4$

(**Note:** $t^2 + 1 \geq 0$ for all t.)

Domain: $(-\infty, \infty)$

Range: $(-\infty, 1) \cup [2, \infty)$

29. $f(x) = \begin{cases} |x| + 1, & x < 1 \\ -x + 1, & x \geq 1 \end{cases}$

(a) $f(-3) = |-3| + 1 = 4$

(b) $f(1) = -1 + 1 = 0$

(c) $f(3) = -3 + 1 = -2$

(d) $f(b^2 + 1) = -(b^2 + 1) + 1 = -b^2$

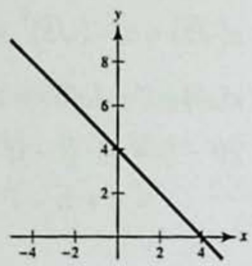
Domain: $(-\infty, \infty)$

Range: $(-\infty, 0] \cup [1, \infty)$

31. $f(x) = 4 - x$

Domain: $(-\infty, \infty)$

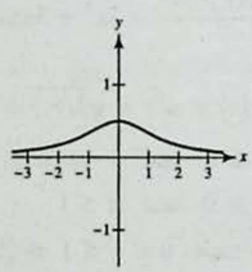
Range: $(-\infty, \infty)$



33. $g(x) = \dfrac{1}{x^2 + 2}$

Domain: $(-\infty, \infty)$

Range: $\left(0, \frac{1}{2}\right]$

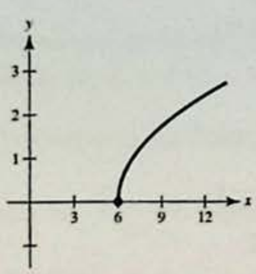


35. $h(x) = \sqrt{x - 6}$

Domain: $x - 6 \geq 0$

$x \geq 6 \Rightarrow [6, \infty)$

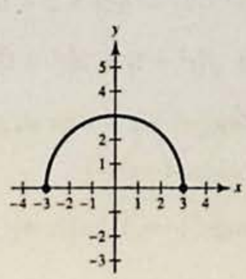
Range: $[0, \infty)$



37. $f(x) = \sqrt{9 - x^2}$

Domain: $[-3, 3]$

Range: $[0, 3]$



39. $x - y^2 = 0 \Rightarrow y = \pm\sqrt{x}$

y is not a function of x. Some vertical lines intersect the graph twice.

41. y is a function of x. Vertical lines intersect the graph at most once.

43. $x^2 + y^2 = 16 \Rightarrow y = \pm\sqrt{16 - x^2}$

y is not a function of x because there are two values of y for some x.

45. $y^2 = x^2 - 1 \Rightarrow y = \pm\sqrt{x^2 - 1}$

y is not a function of x because there are two values of y for some x.

47. The transformation is a horizontal shift two units to the right of the function $f(x) = \sqrt{x}$.

Shifted function: $y = \sqrt{x - 2}$

49. The transformation is a horizontal shift 2 units to the right and a vertical shift 1 unit downward of the function $f(x) = x^2$.

Shifted function: $y = (x - 2)^2 - 1$

51. $y = f(x + 5)$ is a horizontal shift 5 units to the left. Matches d.

52. $y = f(x) - 5$ is a vertical shift 5 units downward. Matches b.

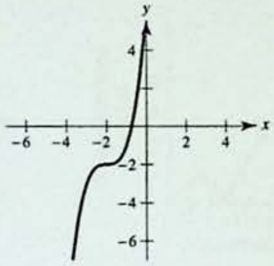
53. $y = -f(-x) - 2$ is a reflection in the y-axis, a reflection in the x-axis, and a vertical shift downward 2 units. Matches c.

54. $y = -f(x - 4)$ is a horizontal shift 4 units to the right, followed by a reflection in the x-axis. Matches a.

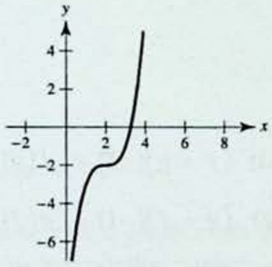
55. $y = f(x + 6) + 2$ is a horizontal shift to the left 6 units, and a vertical shift upward 2 units. Matches e.

56. $y = f(x - 1) + 3$ is a horizontal shift to the right 1 unit, and a vertical shift upward 3 units. Matches g.

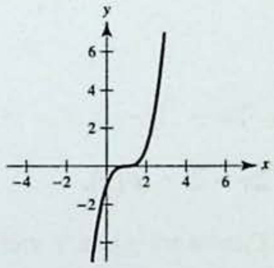
57. (a) The graph is shifted 3 units to the left.



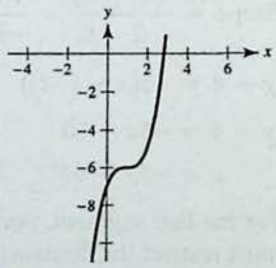
(b) The graph is shifted 1 unit to the right.



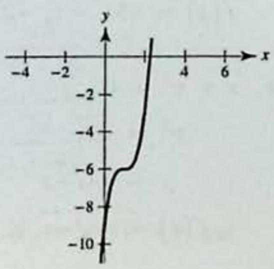
(c) The graph is shifted 2 units upward.



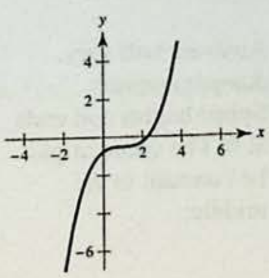
(d) The graph is shifted 4 units downward.



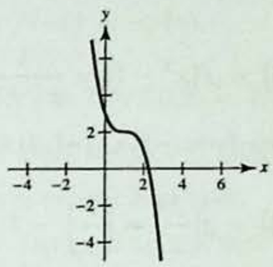
(e) The graph is stretched vertically by a factor of 3.



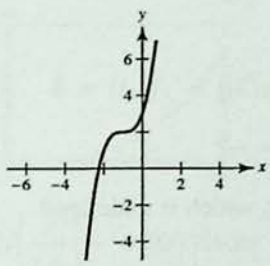
(f) The graph is stretched vertically by a factor of $\frac{1}{4}$.



(g) The graph is a reflection in the x-axis.



(h) The graph is a reflection about the origin.



59. $f(x) = 2x - 5, g(x) = 4 - 3x$

(a) $f(x) + g(x) = (2x - 5) + (4 - 3x) = -x - 1$

(b) $f(x) - g(x) = (2x - 5) - (4 - 3x) = 5x - 9$

(c) $f(x) \cdot g(x) = (2x - 5)(4 - 3x)$

$= -6x^2 + 8x + 15x - 20$

$= -6x^2 + 23x - 20$

(d) $\dfrac{f(x)}{g(x)} = \dfrac{2x - 5}{4 - 3x}$

61. (a) $f(g(1)) = f(0) = 0$

(b) $g(f(1)) = g(1) = 0$

(c) $g(f(0)) = g(0) = -1$

(d) $f(g(-4)) = f(15) = \sqrt{15}$

(e) $f(g(x)) = f(x^2 - 1) = \sqrt{x^2 - 1}$

(f) $g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 - 1 = x - 1, (x \geq 0)$

63. $f(x) = x^2, g(x) = \sqrt{x}$

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x, x \geq 0$

Domain: $[0, \infty)$

$(g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{x^2} = |x|$

Domain: $(-\infty, \infty)$

No. Their domains are different. $(f \circ g) = (g \circ f)$ for $x \geq 0$.

65. $f(x) = \frac{3}{x}$, $g(x) = x^2 - 1$

$(f \circ g)(x) = f(g(x)) = f(x^2 - 1) = \frac{3}{x^2 - 1}$

Domain: all $x \neq \pm 1 \Rightarrow (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

$(g \circ f)(x) = g(f(x)) = g\left(\frac{3}{x}\right) = \left(\frac{3}{x}\right)^2 - 1 = \frac{9}{x^2} - 1 = \frac{9 - x^2}{x^2}$

Domain: all $x \neq 0 \Rightarrow (-\infty, 0) \cup (0, \infty)$

$f \circ g \neq g \circ f$

67. (a) $(f \circ g)(3) = f(g(3)) = f(-1) = 4$

(b) $g(f(2)) = g(1) = -2$

(c) $g(f(5)) = g(-5)$, which is undefined

(d) $(f \circ g)(-3) = f(g(-3)) = f(-2) = 3$

(e) $(g \circ f)(-1) = g(f(-1)) = g(4) = 2$

(f) $f(g(-1)) = f(-4)$, which is undefined

69. $F(x) = \sqrt{2x - 2}$

Let $h(x) = 2x$, $g(x) = x - 2$ and $f(x) = \sqrt{x}$.

Then $(f \circ g \circ h)(x) = f(g(2x)) = f((2x) - 2) = \sqrt{(2x) - 2} = \sqrt{2x - 2} = F(x)$.

(Other answers possible.)

71. (a) If f is even, then $\left(\frac{3}{2}, 4\right)$ is on the graph.

(b) If f is odd, then $\left(\frac{3}{2}, -4\right)$ is on the graph.

73. f is even because the graph is symmetric about the y-axis. g is neither even nor odd. h is odd because the graph is symmetric about the origin.

75. $f(x) = x^2(4 - x^2)$

$f(-x) = (-x)^2(4 - (-x)^2) = x^2(4 - x^2) = f(x)$

f is even.

$f(x) = x^2(4 - x^2) = 0$

$x^2(2 - x)(2 + x) = 0$

Zeros: $x = 0, -2, 2$

77. $f(x) = 2\sqrt[6]{x}$

The domain of f is $x \geq 0$ and the range is $y \geq 0$. Hence, the function is neither even nor odd. The only zero is $x = 0$.

79. Slope $= \frac{4 - (-6)}{-2 - 0} = \frac{10}{-2} = -5$

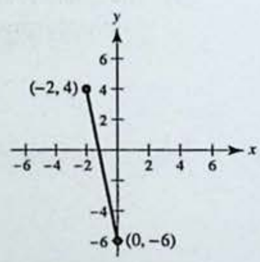
$y - 4 = -5(x - (-2))$

$y - 4 = -5x - 10$

$y = -5x - 6$

For the line segment, you must restrict the domain.

$f(x) = -5x - 6, \ -2 \leq x \leq 0$

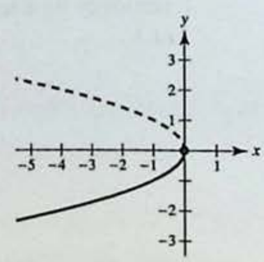


81. $x + y^2 = 0$

$y^2 = -x$

$y = -\sqrt{-x}$

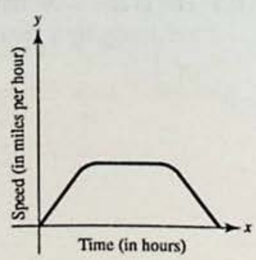
$f(x) = -\sqrt{-x}, \ x \leq 0$



83. Answers will vary.

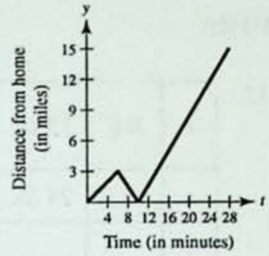
Sample answer:

Speed begins and ends at 0. The speed might be constant in the middle:



85. Answers will vary.
Sample answer:

Distance begins at 0, then the graph has a sharp turn after a few minutes and goes back to 0. Then the graph goes back upward steeply.



87.
$$y = \sqrt{c - x^2}$$
$$y^2 = c - x^2$$
$$x^2 + y^2 = c, \text{ a circle.}$$

For the domain to be $[-5, 5]$, $c = 25$.

89. No. If a horizontal line intersects the graph more than once, then there is more than one x-value corresponding to the same y-value.

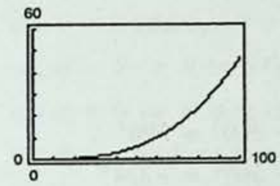
91. No. For example, $y = x^3 + x + 2$ is not odd since $f(-x) \neq -f(x)$.

93. (a) $T(4) = 16°$, $T(15) \approx 23°$

(b) If $H(t) = T(t - 1)$, then the changes in temperature will occur 1 hour later.

(c) If $H(t) = T(t) - 1$, then the overall temperature would be 1 degree lower.

95. (a)



(b) $H\left(\dfrac{x}{1.6}\right) = 0.00004636\left(\dfrac{x}{1.6}\right)^3 \approx 0.00001132x^3$

97. $f(-x) = a_{2n+1}(-x)^{2n+1} + \cdots + a_3(-x)^3 + a_1(-x) = -\left[a_{2n+1}x^{2n+1} + \cdots + a_3x^3 + a_1x\right] = -f(x)$

Odd

99. Let $F(x) = f(x)g(x)$ where f and g are even. Then $F(-x) = f(-x)g(-x) = f(x)g(x) = F(x)$.

So, $F(x)$ is even. Let $F(x) = f(x)g(x)$ where f and g are odd. Then

$F(-x) = f(-x)g(-x) = [-f(x)][-g(x)] = f(x)g(x) = F(x)$.

So, $F(x)$ is even.

101. By equating slopes, $\dfrac{y-2}{0-3} = \dfrac{0-2}{x-3}$

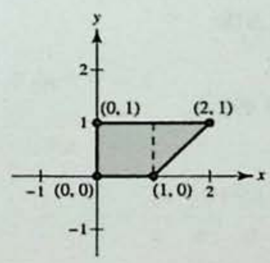
$$y - 2 = \frac{6}{x-3}$$
$$y = \frac{6}{x-3} + 2 = \frac{2x}{x-3},$$
$$L = \sqrt{x^2 + y^2} = \sqrt{x^2 + \left(\frac{2x}{x-3}\right)^2}.$$

103. False. If $f(x) = x^2$, then $f(-3) = f(3) = 9$, but $-3 \neq 3$.

105. True. The function is even.

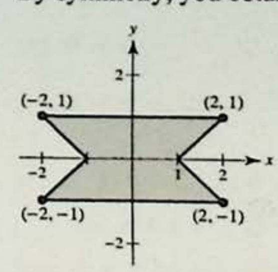
107. False. The constant function $f(x) = 0$ has symmetry with respect to the x-axis.

109. First consider the portion of R in the first quadrant: $x \geq 0$, $0 \leq y \leq 1$ and $x - y \leq 1$; shown below.



The area of this region is $1 + \frac{1}{2} = \frac{3}{2}$.

By symmetry, you obtain the entire region R:



The area of R is $4\left(\frac{3}{2}\right) = 6$.

Section 1.4 Review of Trigonometric Functions

1. In general, if θ is any angle measured in degrees, then the angle $\theta + n(360°)$, n a nonzero integer, is coterminal with θ.

3. $\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{7}{25}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{24}{25}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{7}{24}$

5. (a) $\theta + 360° = 36° + 360° = 396°$

$\theta - 360° = 36° - 360° = -324°$

(b) $\theta + 360° = -120° + 360° = 240°$

$\theta - 360° = -120° - 360° = -480°$

7. (a) $\theta + 2\pi = \dfrac{\pi}{9} + 2\pi = \dfrac{19\pi}{9}$

$\theta - 2\pi = \dfrac{\pi}{9} - 2\pi = -\dfrac{17\pi}{9}$

(b) $\theta + 2\pi = \dfrac{4\pi}{3} + 2\pi = \dfrac{10\pi}{3}$

$\theta - 2\pi = \dfrac{4\pi}{3} - 2\pi = -\dfrac{2\pi}{3}$

9. (a) $30°\left(\dfrac{\pi}{180°}\right) = \dfrac{\pi}{6} \approx 0.524$

(b) $150°\left(\dfrac{\pi}{180°}\right) = \dfrac{5\pi}{6} \approx 2.618$

(c) $315°\left(\dfrac{\pi}{180°}\right) = \dfrac{7\pi}{4} \approx 5.498$

(d) $120°\left(\dfrac{\pi}{180°}\right) = \dfrac{2\pi}{3} \approx 2.094$

11. (a) $\dfrac{3\pi}{2}\left(\dfrac{180°}{\pi}\right) = 270°$

(b) $\dfrac{7\pi}{6}\left(\dfrac{180°}{\pi}\right) = 210°$

(c) $-\dfrac{7\pi}{12}\left(\dfrac{180°}{\pi}\right) = -105°$

(d) $-2.367\left(\dfrac{180°}{\pi}\right) \approx -135.62°$

13.

r	8 ft	15 in.	85 cm	24 in.	$\dfrac{12{,}963}{\pi}$ mi
s	12 ft	24 in.	63.75π cm	96 in.	8642 mi
θ	1.5	1.6	$\dfrac{3\pi}{4}$	4	$\dfrac{2\pi}{3}$

15. (a) $x = 3, y = 4, r = 5$

$\sin\theta = \frac{4}{5}$ $\quad\csc\theta = \frac{5}{4}$

$\cos\theta = \frac{3}{5}$ $\quad\sec\theta = \frac{5}{3}$

$\tan\theta = \frac{4}{3}$ $\quad\cot\theta = \frac{3}{4}$

(b) $x = -12, y = -5, r = 13$

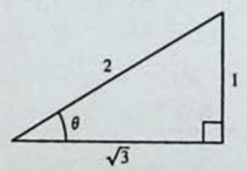
$\sin\theta = -\frac{5}{13}$ $\quad\csc\theta = -\frac{13}{5}$

$\cos\theta = -\frac{12}{13}$ $\quad\sec\theta = -\frac{13}{12}$

$\tan\theta = \frac{5}{12}$ $\quad\cot\theta = \frac{12}{5}$

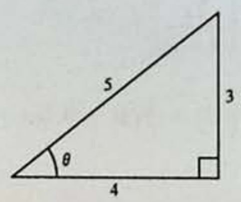
17. $x^2 + 1^2 = 2^2 \Rightarrow x = \sqrt{3}$

$\cos\theta = \dfrac{x}{2} = \dfrac{\sqrt{3}}{2}$



19. $4^2 + y^2 = 5^2 \Rightarrow y = 3$

$\cot\theta = \dfrac{4}{y} = \dfrac{4}{3}$



21. (a) $\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{1}{2}$

$\tan 60° = \sqrt{3}$

(b) $\sin 120° = \sin 60° = \frac{\sqrt{3}}{2}$

$\cos 120° = -\cos 60° = -\frac{1}{2}$

$\tan 120° = -\tan 60° = -\sqrt{3}$

(c) $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\tan \frac{\pi}{4} = 1$

(d) $\sin \frac{5\pi}{4} - \sin \frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$\cos \frac{5\pi}{4} = \cos \frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$\tan \frac{5\pi}{4} = \tan \frac{\pi}{4} = 1$

23. (a) $\sin 225° = -\sin 45° = -\frac{\sqrt{2}}{2}$

$\cos 225° = -\cos 45° = -\frac{\sqrt{2}}{2}$

$\tan 225° = \tan 45° = 1$

(b) $\sin(-225°) = \sin 45° = \frac{\sqrt{2}}{2}$

$\cos(-225°) = -\cos 45° = -\frac{\sqrt{2}}{2}$

$\tan(-225°) = -\tan 45° = -1$

(c) $\sin\frac{5\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

$\cos\frac{5\pi}{3} = \cos\frac{\pi}{3} = \frac{1}{2}$

$\tan\frac{5\pi}{3} = -\tan\frac{\pi}{3} = -\sqrt{3}$

(d) $\sin\frac{11\pi}{6} = -\sin\frac{\pi}{6} = -\frac{1}{2}$

$\cos\frac{11\pi}{6} = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\tan\frac{11\pi}{6} = -\tan\frac{\pi}{6} = -\frac{\sqrt{3}}{3}$

25. (a) $\sin 10° \approx 0.1736$

(b) $\csc 10° \approx 5.759$

27. (a) $\tan\frac{\pi}{9} \approx 0.3640$

(b) $\tan\frac{10\pi}{9} \approx 0.3640$

29. (a) $\sin\theta < 0 \Rightarrow \theta$ is in Quadrant III or IV.

$\cos\theta < 0 \Rightarrow \theta$ is in Quadrant II or III.

$\sin\theta < 0$ **and** $\cos\theta < 0 \Rightarrow \theta$ is in Quadrant III.

(b) $\sec\theta > 0 \Rightarrow \theta$ is in Quadrant I or IV.

$\cot\theta < 0 \Rightarrow \theta$ is in Quadrant II or IV.

$\sec\theta > 0$ **and** $\cot\theta < 0 \Rightarrow \theta$ is in Quadrant IV.

31. (a) $\cos\theta = \frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}, \frac{7\pi}{4}$

(b) $\cos\theta = -\frac{\sqrt{2}}{2}$

$\theta = \frac{3\pi}{4}, \frac{5\pi}{4}$

33. (a) $\tan\theta = 1$

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

(b) $\cot\theta = -\sqrt{3}$

$\theta = \frac{5\pi}{6}, \frac{11\pi}{6}$

35. $2\sin^2\theta = 1$

$\sin\theta = \pm\frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

37. $\tan^2\theta - \tan\theta = 0$

$\tan\theta(\tan\theta - 1) = 0$

$\tan\theta = 0$ $\qquad$ $\tan\theta = 1$

$\theta = 0, \pi, 2\pi$ $\qquad$ $\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

39. $\sec\theta\csc\theta - 2\csc\theta = 0$

$\csc\theta(\sec\theta - 2) = 0$

($\csc\theta \neq 0$ for any value of θ)

$\sec\theta = 2$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

41.

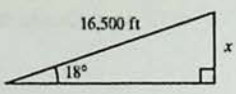
$$\cos^2 \theta + \sin \theta = 1$$
$$1 - \sin^2 \theta + \sin \theta = 1$$
$$\sin^2 \theta - \sin \theta = 0$$
$$\sin \theta(\sin \theta - 1) = 0$$

$\sin \theta = 0$ $\qquad$ $\sin \theta = 1$

$\theta = 0, \pi, 2\pi$ $\qquad$ $\theta = \dfrac{\pi}{2}$

43. $(275 \text{ ft/sec})(60 \text{ sec}) = 16{,}500$ feet

$$\sin 18° = \frac{a}{16{,}500}$$
$$a = 16{,}500 \sin 18° \approx 5099 \text{ feet}$$



45. $y = 2 \sin 2x$

$$\text{Period} = \frac{2\pi}{2} = \pi$$

$$\text{Amplitude} = |2| = 2$$

47. $y = -3 \sin 4\pi x$

$$\text{Period} = \frac{2\pi}{4\pi} = \frac{1}{2}$$

$$\text{Amplitude} = |-3| = 3$$

49. $y = 5 \tan 2x$

$$\text{Period} = \frac{\pi}{2}$$

51. $y = \sec 5x$

$$\text{Period} = \frac{2\pi}{5}$$

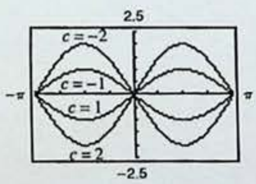
53. (a) $f(x) = c \sin x$; changing c changes the amplitude.

When $c = -2$: $f(x) = -2 \sin x$.

When $c = -1$: $f(x) = -\sin x$.

When $c = 1$: $f(x) = \sin x$.

When $c = 2$: $f(x) = 2 \sin x$.



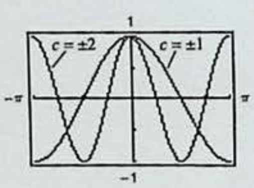
(b) $f(x) = \cos(cx)$; changing c changes the period.

When $c = -2$: $f(x) = \cos(-2x) = \cos 2x$.

When $c = -1$: $f(x) = \cos(-x) = \cos x$.

When $c = 1$: $f(x) = \cos x$.

When $c = 2$: $f(x) = \cos 2x$.



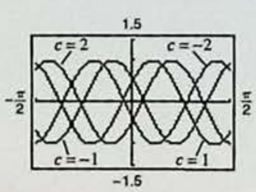
(c) $f(x) = \cos(\pi x - c)$; changing c causes a horizontal shift.

When $c = -2$: $f(x) = \cos(\pi x + 2)$.

When $c = -1$: $f(x) = \cos(\pi x + 1)$.

When $c = 1$: $f(x) = \cos(\pi x - 1)$.

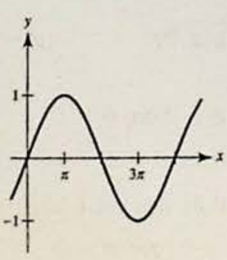
When $c = 2$: $f(x) = \cos(\pi x - 2)$.



55. $y = \sin \dfrac{x}{2}$

Period: 4π

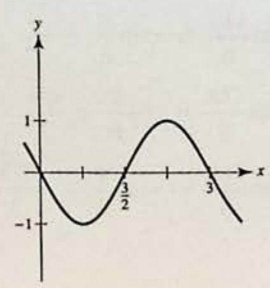
Amplitude: 1



57. $y = -\sin \dfrac{2\pi x}{3}$

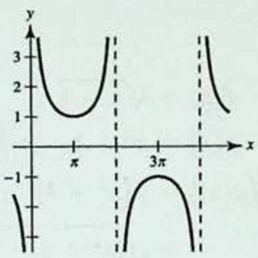
Period: 3

Amplitude: 1



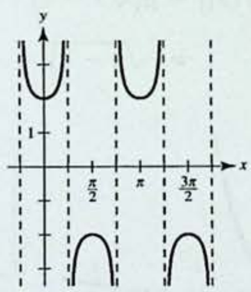
59. $y = \csc\frac{x}{2}$

Period: 4π



61. $y = 2\sec 2x$

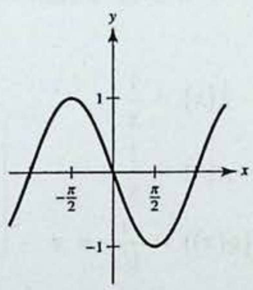
Period: π



63. $y = \sin(x + \pi)$

Period: 2π

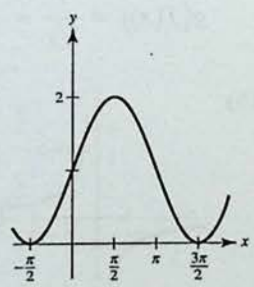
Amplitude: 1



65. $y = 1 + \cos\left(x - \frac{\pi}{2}\right)$

Period: 2π

Amplitude: 1



67. $y = a\cos(bx - c)$

From the graph, we see that the amplitude is 3, the period is 4π, and the horizontal shift is π. Thus,

$$a = 3$$

$$\frac{2\pi}{b} = 4\pi \Rightarrow b = \frac{1}{2}$$

$$\frac{c}{b} = \pi \Rightarrow c = \frac{\pi}{2}.$$

Therefore, $y = 3\cos\left[(1/2)x - (\pi/2)\right]$.

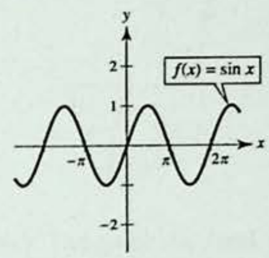
69. Yes. Use the right-triangle definitions of the trigonometric functions.

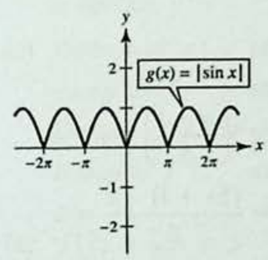
71. The range of the cosine function is $[-1, 1]$. The range of the secant function is $(-\infty, -1] \cup [1, \infty)$.

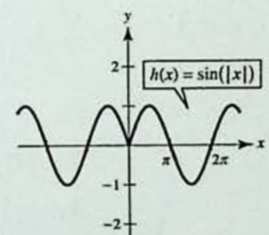
73. $f(x) = \sin x$

$g(x) = |\sin x|$

$h(x) = \sin|x|$



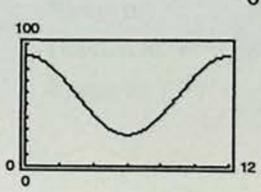




The graph of $|f(x)|$ will reflect any parts of the graph of $f(x)$ below the x-axis about the y-axis.

The graph of $f(|x|)$ will reflect the part of the graph of $f(x)$ to the right of the y-axis about the y-axis.

75. $S = 58.3 + 32.5\cos\frac{\pi t}{6}$



Sales exceed 75,000 during the months of January, November, and December.

77. False. 4π radians (not 4 radians) corresponds to two complete revolutions from the initial side to the terminal side of an angle.

79. False. The amplitude of the function $y = \frac{1}{2}\sin 2x$ is one-half the amplitude of the function $y = \sin x$.

Section 1.5 Inverse Functions

1. The graphs of f and f^{-1} are mirror images with respect to the line $y = x$.

3. $\arccos x$ is the angle θ whose cosine is x, where $0 \le \theta \le \pi$.

5. Matches (c)

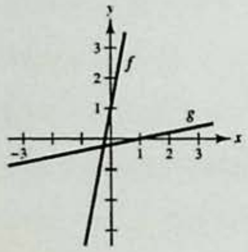
6. Matches (b)

7. Matches (a)

8. Matches (d)

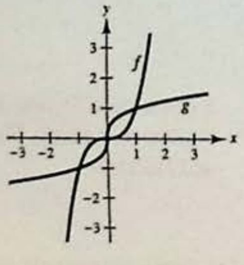
9. (a)
$$f(x) = 5x + 1$$
$$g(x) = \frac{x-1}{5}$$
$$f(g(x)) = f\left(\frac{x-1}{5}\right) = 5\left(\frac{x-1}{5}\right) + 1 = x$$
$$g(f(x)) = g(5x+1) = \frac{(5x+1)-1}{5} = x$$

(b)



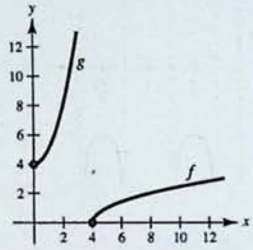
11. (a)
$$f(x) = x^3$$
$$g(x) = \sqrt[3]{x}$$
$$f(g(x)) = f\left(\sqrt[3]{x}\right) = \left(\sqrt[3]{x}\right)^3 = x$$
$$g(f(x)) = g(x^3) = \sqrt[3]{x^3} = x$$

(b)



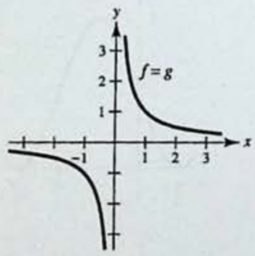
13. (a)
$$f(x) = \sqrt{x-4}$$
$$g(x) = x^2 + 4, \quad x \ge 0$$
$$f(g(x)) = f(x^2+4) = \sqrt{(x^2+4)-4} = \sqrt{x^2} = x$$
$$g(f(x)) = g\left(\sqrt{x-4}\right) = \left(\sqrt{x-4}\right)^2 + 4 = x - 4 + 4 = x$$

(b)



15. (a)
$$f(x) = \frac{1}{x}$$
$$g(x) = \frac{1}{x}$$
$$f(g(x)) = \frac{1}{1/x} = x$$
$$g(f(x)) = \frac{1}{1/x} = x$$

(b)



17. $f(\theta) = \sin\theta$

Not one-to-one; does not have an inverse

19. $f(x) = 2 - x - x^3$

One-to-one; has an inverse

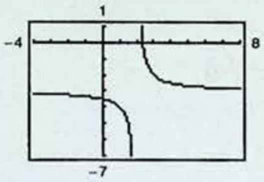
21. $f(x) = \dfrac{1}{3x+1}$

One-to-one; has an inverse

23. $f(x) = \tan 2\pi x$

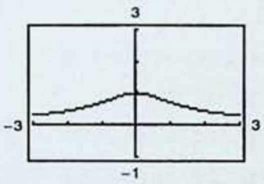
Not one-to-one; does not have an inverse

25. $h(s) = \dfrac{1}{s-2} - 3$



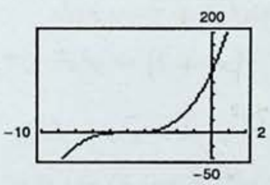
One-to-one; has an inverse

27. $g(t) = \dfrac{1}{\sqrt{t^2+1}}$



Not one-to-one; does not have an inverse

29. $g(x) = (x+5)^3$



One-to-one; has an inverse

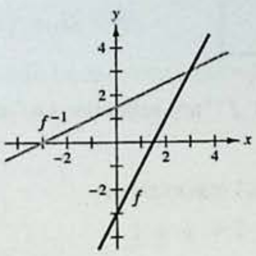
31. (a) $f(x) = 2x - 3 = y$

$$x = \frac{y+3}{2}$$

$$y = \frac{x+3}{2}$$

$$f^{-1}(x) = \frac{x+3}{2}$$

(b)



(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: all real numbers
Range of f: all real numbers
Domain of f^{-1}: all real numbers
Range of f^{-1}: all real numbers

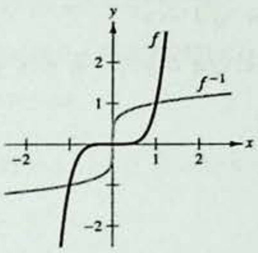
33. (a) $f(x) = x^5 = y$

$$x = \sqrt[5]{y}$$

$$y = \sqrt[5]{x}$$

$$f^{-1}(x) = \sqrt[5]{x} = x^{1/5}$$

(b)



(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: all real numbers
Range of f: all real numbers
Domain of f^{-1}: all real numbers
Range of f^{-1}: all real numbers

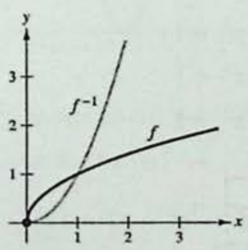
35. (a) $f(x) = \sqrt{x} = y$

$$x = y^2$$

$$y = x^2$$

$$f^{-1}(x) = x^2, \quad x \ge 0$$

(b)



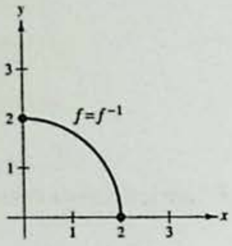
(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: $x \ge 0$
Range of f: $y \ge 0$
Domain of f^{-1}: $x \ge 0$
Range of f^{-1}: $y \ge 0$

37. (a) $f(x) = \sqrt{4 - x^2} = y, \quad 0 \le x \le 2$

$$4 - x^2 = y^2$$
$$x^2 = 4 - y^2$$
$$x = \sqrt{4 - y^2}$$
$$y = \sqrt{4 - x^2}$$
$$f^{-1}(x) = \sqrt{4 - x^2}, \quad 0 \le x \le 2$$

(b)



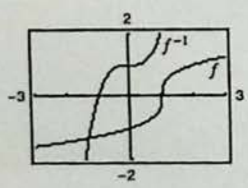
(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$. In fact, the graphs are identical.

(d) Domain of f: $0 \le x \le 2$
Range of f: $0 \le y \le 2$
Domain of f^{-1}: $0 \le x \le 2$
Range of f^{-1}: $0 \le y \le 2$

39. (a) $f(x) = \sqrt[3]{x - 1} = y$

$$x - 1 = y^3$$
$$x = y^3 + 1$$
$$y = x^3 + 1$$
$$f^{-1}(x) = x^3 + 1$$

(b)



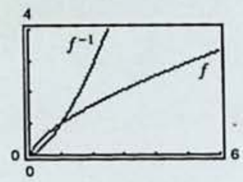
(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: all real numbers
Range of f: all real numbers
Domain of f^{-1}: all real numbers
Range of f^{-1}: all real numbers

41. (a) $f(x) = x^{2/3} = y, \quad x \ge 0$

$$x = y^{3/2}$$
$$y = x^{3/2}$$
$$f^{-1}(x) = x^{3/2}, \quad x \ge 0$$

(b)



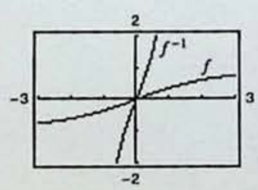
(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: $x \ge 0$
Range of f: $y \ge 0$
Domain of f^{-1}: $x \ge 0$
Range of f^{-1}: $y \ge 0$

43. (a) $f(x) = \dfrac{x}{\sqrt{x^2 + 7}} = y$

$$x = y\sqrt{x^2 + 7}$$
$$x^2 = y^2(x^2 + 7) = y^2x^2 + 7y^2$$
$$x^2(1 - y^2) = 7y^2$$
$$x = \frac{\sqrt{7}y}{\sqrt{1 - y^2}}$$
$$y = \frac{\sqrt{7}x}{\sqrt{1 - x^2}}$$
$$f^{-1}(x) = \frac{\sqrt{7}x}{\sqrt{1 - x^2}}, \quad -1 < x < 1$$

(b)



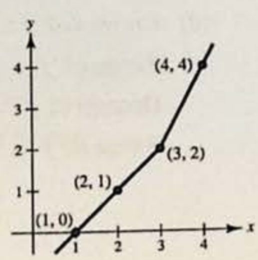
(c) The graphs of f and f^{-1} are reflections of each other in the line $y = x$.

(d) Domain of f: all real numbers
Range of f: $-1 < y < 1$
Domain of f^{-1}: $-1 < x < 1$
Range of f^{-1}: all real numbers

45.

x	0	1	2	4
$f(x)$	1	2	3	4

x	1	2	3	4
$f^{-1}(x)$	0	1	2	4



47. (a) Let x be the number of pounds of the commodity costing \$1.25 per pound. Because there are 50 pounds total, the amount of the second commodity is $50 - x$. The total cost is

$$f(x) = y = 1.25x + 2.75(50 - x)$$
$$= -1.5x + 137.5, \; 0 \le x \le 50$$

(b)
$$y = -1.5x + 137.5$$
$$1.5x = 137.5 - y$$
$$x = \frac{(137.5 - y)}{1.5}$$
$$y = f^{-1}(x) = \frac{2}{3}(137.5 - x)$$

x represents the total cost and y represents the number of pounds of the less expensive commodity.

(c) The range of f is $[62.5, 137.5]$, so the domain of f^{-1} is the same. $50(1.25) = 62.5$ gives the total cost when purchasing 50 pounds of the less expensive commodity, and $50(2.75) = 137.5$ gives the total cost when purchasing 50 pounds of the more expensive commodity.

(d) If $x = 73$, then $f^{-1}(73) = 43$ pounds.

49. $f(x) = \sqrt{x - 2}, \; x \ge 2$

f is one-to-one; has an inverse.

$$y = \sqrt{x - 2}, \; x \ge 2, \; y \ge 0$$
$$y^2 = x - 2$$
$$x = y^2 + 2$$
$$f^{-1}(x) = x^2 + 2, \; x \ge 0$$

51. $f(x) = -3$

Not one-to-one; does not have an inverse.

53. $f(x) = ax + b$

f is one-to-one; has an inverse.

$$ax + b = y$$
$$x = \frac{y - b}{a}$$
$$y = \frac{x - b}{a}$$
$$f^{-1}(x) = \frac{x - b}{a}, \quad a \ne 0$$

55. $f(x) = (x - 4)^2$ on $[4, \infty)$

f passes the Horizontal Line Test on $[4, \infty)$, so it is one-to-one.

57. $f(x) = \dfrac{4}{x^2}$ on $(0, \infty)$

f passes the Horizontal Line Test on $(0, \infty)$, so it is one-to-one.

59. $f(x) = \cos x$ on $[0, \pi]$

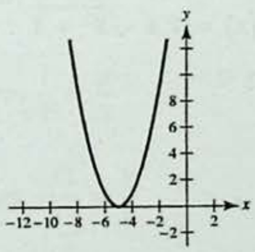
f passes the Horizontal Line Test on $[0, \pi]$, so it is one-to-one.

61. $f(x) = (x - 3)^2$ is one-to-one for $x \ge 3$.

$$(x - 3)^2 = y$$
$$x - 3 = \sqrt{y}$$
$$x = \sqrt{y} + 3$$
$$y = \sqrt{x} + 3$$
$$f^{-1}(x) = \sqrt{x} + 3, \quad x \ge 0$$

(Answer is not unique.)

63. (a) $f(x) = (x + 5)^2$

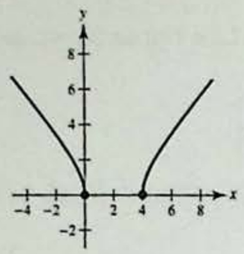


(b) f is one-to-one on $[-5, \infty)$. (Note that f is also one-to-one on $(-\infty, -5]$.)

(c)
$$f(x) = (x + 5)^2 = y, \qquad x \ge -5$$
$$x + 5 = \sqrt{y}$$
$$x = \sqrt{y} - 5$$
$$y = \sqrt{x} - 5$$
$$f^{-1}(x) = \sqrt{x} - 5$$

(d) Domain of f^{-1}: $x \ge 0$

65. (a) $f(x) = \sqrt{x^2 - 4x}$

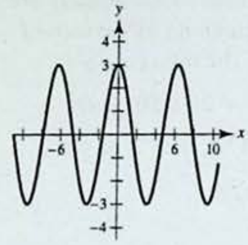


(b) f is one-to-one on $[4, \infty)$. (Note that f is also one-to-one on $(-\infty, 0]$.)

(c)
$$\begin{aligned} f(x) = \sqrt{x^2 - 4x} &= y,\ x \geq 4 \\ x^2 - 4x &= y^2 \\ x^2 - 4x + 4 &= y^2 + 4 \\ (x-2)^2 &= y^2 + 4 \\ x - 2 &= \sqrt{y^2 + 4} \\ x &= 2 + \sqrt{y^2 + 4} \\ y &= 2 + \sqrt{x^2 + 4} \\ f^{-1}(x) &= 2 + \sqrt{x^2 + 4} \end{aligned}$$

(d) Domain of $f^{-1}: x \geq 0$

67. (a) $f(x) = 3\cos x$



(b) f is one-to-one on $[0, \pi]$. (other answers possible)

(c)
$$\begin{aligned} f(x) = 3\cos x &= y \\ \cos x &= \frac{y}{3} \\ x &= \arccos\left(\frac{y}{3}\right) \\ y &= \arccos\left(\frac{x}{3}\right) \\ f^{-1}(x) &= \arccos\left(\frac{x}{3}\right) \end{aligned}$$

(d) Domain of $f^{-1}: -3 \leq x \leq 3$

69. $f(x) = x^3 + 2x - 1$

$f(1) = 2 = a \Rightarrow f^{-1}(2) = 1$

71. $f(x) = 5\sin x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

$$f\left(-\frac{\pi}{6}\right) = 5\sin\left(-\frac{\pi}{6}\right) = 5\left(-\frac{1}{2}\right) = -\frac{5}{2} = a \Rightarrow f^{-1}\left(-\frac{5}{2}\right) = -\frac{\pi}{6}$$

73. $f(x) = x^3 - \frac{4}{x}$

$f(2) = 6 = a \Rightarrow f^{-1}(6) = 2$

In Exercises 75–77, use the following.

$f(x) = \frac{1}{8}x - 3$ **and** $g(x) = x^3$

$f^{-1}(x) = 8(x + 3)$ **and** $g^{-1}(x) = \sqrt[3]{x}$

75. $(f^{-1} \circ g^{-1})(1) = f^{-1}(g^{-1}(1)) = f^{-1}(1) = 32$

77. $(f^{-1} \circ f^{-1})(-2) = f^{-1}(f^{-1}(-2)) = f^{-1}(8) = 88$

In Exercises 79–81, use the following.

$f(x) = x + 4$ **and** $g(x) = 2 - x^3$

$f^{-1}(x) = x - 4$ **and** $g^{-1}(x) = \sqrt[3]{2 - x}$

79.
$$\begin{aligned} (g^{-1} \circ f^{-1})(x) &= g^{-1}(f^{-1}(x)) \\ &= g^{-1}(x - 4) \\ &= \sqrt[3]{2 - (x - 4)} \\ &= \sqrt[3]{6 - x} \end{aligned}$$

81.
$$\begin{aligned} (f \circ g)(x) &= f(g(x)) \\ &= f(2 - x^3) \\ &= (2 - x^3) + 4 \\ &= 6 - x^3 \end{aligned}$$

So, $(f \circ g)^{-1}(x) = \sqrt[3]{6 - x}$.

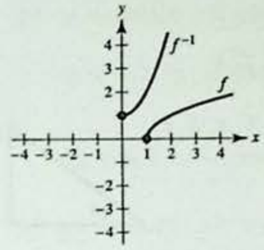
Note: $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$

83. (a) f is one-to-one because it passes the Horizontal Line Test.

(b) The domain of f^{-1} is the range of f: $[-2, 2]$.

(c) $f^{-1}(2) = -4$ because $f(-4) = 2$.

85.

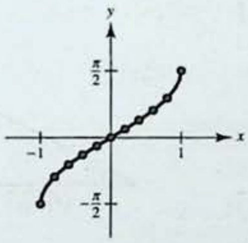


87. $y = \arcsin x$

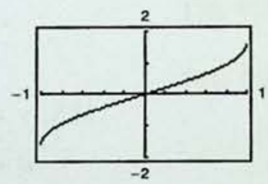
(a)

x	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1
y	-1.571	-0.927	-0.644	-0.412	-0.201	0	0.201	0.412	0.644	0.927	1.571

(b)



(c)



(d) Symmetric about origin: $\arcsin(-x) = -\arcsin x$

Intercept: $(0, 0)$

89. $y = \arccos x$

$\left(-\frac{\sqrt{2}}{2}, \frac{3\pi}{4}\right)$ because $\cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$.

$\left(\frac{1}{2}, \frac{\pi}{3}\right)$ because $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$.

$\left(\frac{\sqrt{3}}{2}, \frac{\pi}{6}\right)$ because $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$.

91. $f(x) = \arcsin(x - 1)$

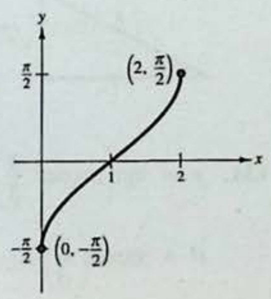
$x - 1 = \sin y$

$x = 1 + \sin y$

Domain: $[0, 2]$

Range: $[-\pi/2, \pi/2]$

$f(x)$ is the graph of $\arcsin x$ shifted right one unit.

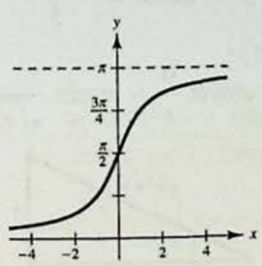


93. $f(x) = \arctan x + \frac{\pi}{2}$

$x = \tan\left(y - \frac{\pi}{2}\right)$

Domain: $(-\infty, \infty)$

Range: $[0, \pi]$



$f(x)$ is the graph of $\arctan x$ shifted $\pi/2$ unit upward.

95. $\arcsin \frac{1}{2} = \frac{\pi}{6}$

97. $\arccos \frac{1}{2} = \frac{\pi}{3}$

99. $\arctan \frac{\sqrt{3}}{3} = \frac{\pi}{6}$

101. $\operatorname{arccsc}\left(-\sqrt{2}\right) = -\frac{\pi}{4}$

103. $\arccos(0.051) \approx 1.52$

105. $\operatorname{arcsec}(1.269) = \arccos\left(\frac{1}{1.269}\right) \approx 0.66$

107. $\cos\left[\arccos(-0.1)\right] = -0.1$

109. No. Graphically, adding a constant shift the graph vertically.

111. The trigonometric functions are not one-to-one. So, their domains must be restricted to define the inverse trigonometric functions.

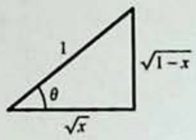
113. $\arcsin(3x - \pi) = \frac{1}{2}$

$3x - \pi = \sin\left(\frac{1}{2}\right)$

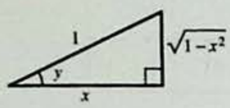
$x = \frac{1}{3}\left[\sin\left(\frac{1}{2}\right) + \pi\right] \approx 1.207$

115. $\arcsin\sqrt{2x} = \arccos\sqrt{x}$

$$\sqrt{2x} = \sin\left(\arccos\sqrt{x}\right)$$
$$\sqrt{2x} = \sqrt{1-x},\ 0 \le x \le 1$$
$$2x = 1 - x$$
$$3x = 1$$
$$x = \tfrac{1}{3}$$



In Exercises 117–121, use the triangle.



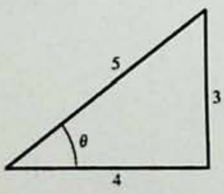
117. $y = \arccos x$

$\cos y = x$

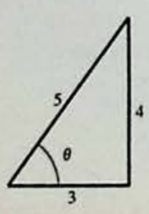
119. $\tan y = \dfrac{\sqrt{1-x^2}}{x}$

121. $\sec y = \dfrac{1}{x}$

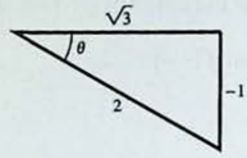
123. (a) $\sin\left(\arctan\dfrac{3}{4}\right) = \dfrac{3}{5}$



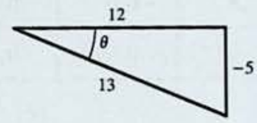
(b) $\sec\left(\arcsin\dfrac{4}{5}\right) = \dfrac{5}{3}$



125. (a) $\cot\left[\arcsin\left(-\dfrac{1}{2}\right)\right] = \cot\left(-\dfrac{\pi}{6}\right) = -\sqrt{3}$

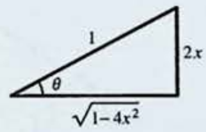


(b) $\csc\left[\arctan\left(-\dfrac{5}{12}\right)\right] = -\dfrac{13}{5}$



127. $y = \cos(\arcsin 2x)$

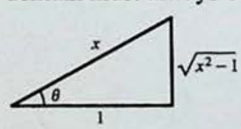
$$\theta = \arcsin 2x$$
$$y = \cos\theta = \sqrt{1-4x^2}$$



129. $y = \sin(\operatorname{arcsec} x)$

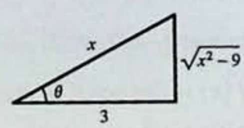
$$\theta = \operatorname{arcsec} x,\ 0 \le \theta \le \pi,\ \theta \ne \frac{\pi}{2}$$
$$y = \sin\theta = \frac{\sqrt{x^2-1}}{|x|}$$

The absolute value bars on x are necessary because of the restriction $0 \le \theta \le \pi$, $\theta \ne \pi/2$, and $\sin\theta$ for this domain must always be nonnegative.

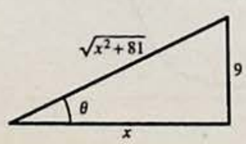


131. $y = \tan\left(\operatorname{arcsec}\dfrac{x}{3}\right)$

$$\theta = \operatorname{arcsec}\frac{x}{3}$$
$$y = \tan\theta = \frac{x^2-9}{3}$$



133. $\arctan\dfrac{9}{x} = \arcsin\dfrac{9}{\sqrt{x^2+81}}$



135. (a) $\operatorname{arccsc} x = \arcsin \frac{1}{x}, |x| \geq 1$

Let $y = \operatorname{arccsc} x$.

Then for $-\frac{\pi}{2} \leq y < 0$ and $0 < y \leq \frac{\pi}{2}$,

$\csc y = x \Rightarrow \sin y = \frac{1}{x}$.

So, $y = \arcsin\left(\frac{1}{x}\right)$. Therefore,

$\operatorname{arccsc} x = \arcsin\left(\frac{1}{x}\right)$.

(b) $\arctan x + \arctan \frac{1}{x} = \frac{\pi}{2}, x > 0$

Let $y = \arctan x + \arctan(1/x)$.

$$\text{Then } \tan y = \frac{\tan(\arctan x) + \tan[\arctan(1/x)]}{1 - \tan(\arctan x)\tan[\arctan(1/x)]}$$

$$= \frac{x + (1/x)}{1 - x(1/x)}$$

$$= \frac{x + (1/x)}{0} \text{ (which is undefined).}$$

So, $y = \pi/2$. Therefore,
$\arctan x + \arctan(1/x) = \pi/2$.

137. (a) $\operatorname{arccot} x = y$ if and only if $\cot y = x$,
$0 < y < \pi$.

For $x > 0$, $\cot y > 0$ and $0 < y < \frac{\pi}{2}$.

So, $\tan y = \frac{1}{x} > 0$ and $y = \arctan\left(\frac{1}{x}\right)$.

For $x = 0$, $\operatorname{arccot}(0) = \frac{\pi}{2}$.

For $x < 0$, $\cot y < 0$ and $\frac{\pi}{2} < y < \pi$.

So, $\tan y = \frac{1}{x} < 0$ and $\arctan\left(\frac{1}{x}\right) < 0$.

Therefore, you need to add π to get

$y = \pi + \arctan\left(\frac{1}{x}\right)$.

(b) $y = \operatorname{arcsec} x$ if and only if $\sec y = x, |x| \geq 1$,

$0 \leq y \leq \pi, y \neq \frac{\pi}{2}$.

So, $\cos y = \frac{1}{x}$ and $y = \arccos\left(\frac{1}{x}\right)$.

(c) $y = \operatorname{arccsc} x$ if and only if $\csc y = x$, $|x| \geq 1$,

$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}, y \neq 0$.

So, $\sin y = \frac{1}{x}$ and $y = \arcsin\left(\frac{1}{x}\right)$.

139. False. Let $f(x) = x^2$.

141. False

$$\arcsin^2 0 + \arccos^2 0 = 0 + \left(\frac{\pi}{2}\right)^2 \neq 1$$

143. True

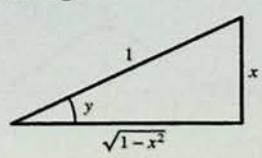
145. Let f and g be one-to-one functions.

Let $(f \circ g)(x) = y$, then $x = (f \circ g)^{-1}(y)$. Also:

$$(f \circ g)(x) = y$$
$$f(g(x)) = y$$
$$g(x) = f^{-1}(y)$$
$$x = g^{-1}(f^{-1}(y))$$
$$x = (g^{-1} \circ f^{-1}(y))$$

So, $(f \circ g)^{-1}(y) = (g^{-1} \circ f^{-1})(y)$ and
$(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

147. Let $y = \sin^{-1} x$. Then $\sin y = x$ and
$\cos(\sin^{-1} x) = \cos(y) = \sqrt{1 - x^2}$, as indicated in the figure.



149. $$\tan(\arctan x + \arctan y) = \frac{\tan(\arctan x + \arctan y)}{1 - \tan(\arctan x)\tan(\arctan y)} = \frac{x + y}{1 - xy}, xy \neq 1$$

So, $\arctan x + \arctan y = \arctan\left(\frac{x + y}{1 - xy}\right), xy \neq 1$.

Let $x = \frac{1}{2}$ and $y = \frac{1}{3}$.

$$\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right) = \arctan \frac{\frac{1}{2} + \frac{1}{3}}{1 - \left(\frac{1}{2} \cdot \frac{1}{3}\right)} = \arctan \frac{\frac{5}{6}}{1 - \frac{1}{6}} = \arctan \frac{\frac{5}{6}}{\frac{5}{6}} = \arctan 1 = \frac{\pi}{4}$$

151. $f(x) = kx + \sin x$

For $k \geq 1$, f is one-to-one, and for $k \leq -1$, f is one-to-one. Therefore, f has an inverse for $k \geq 1$ and $k \leq 1$.

153. f is one-to-one if $f(x_1) = f(x_2)$ implies $x_1 = x_2$. So assume

$$\begin{aligned} f(x_1) &= f(x_2) \\ \frac{ax_1 + b}{cx_1 + d} &= \frac{ax_2 + b}{cx_2 + d} \\ acx_1x_2 + adx_1 + bcx_2 + bd &= acx_1x_2 + adx_2 + bcx_1 + bd \\ adx_1 + bcx_2 &= adx_2 + bcx_1 \\ (ad - bc)x_1 &= (ad - bc)x_2. \end{aligned}$$

So, $x_1 = x_2$ if $ad - bc \neq 0$. To find f^{-1}, solve for x as follows.

$$\begin{aligned} y &= \frac{ax + b}{cx + d} \\ ycx + yd &= ax + b \\ (yc - a)x &= b - yd \\ x &= \frac{b - yd}{yc - a} \\ f^{-1}(x) &= \frac{b - dx}{cx - a} \end{aligned}$$

Section 1.6 Exponential and Logarithmic Functions

1. $f(x) = e^x$. Domain is $(-\infty, \infty)$ and range is $(0, \infty)$. f is continuous, increasing, one-to-one, and concave upwards on its entire domain.

$\lim_{x\to-\infty} e^x = 0$ and $\lim_{x\to\infty} e^x = \infty$

3. The functions $f(x) = e^x$ and $g(x) = \ln x$ are inverses of each other. So, $\ln e^x = g(f(x)) = x$.

5. (a) $25^{3/2} = 5^3 = 125$

(b) $81^{1/2} = 9$

(c) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

(d) $27^{-1/3} = \frac{1}{27^{1/3}} = \frac{1}{3}$

7. (a) $(5^2)(5^3) = 5^{2+3} = 5^5 = 3125$

(b) $(5^2)(5^{-3}) = 5^{2-3} = 5^{-1} = \frac{1}{5}$

(c) $\frac{5^3}{25^2} = \frac{5^3}{5^4} = \frac{1}{5}$

(d) $\left(\frac{1}{4}\right)^2 2^6 - \frac{2^6}{2^4} = 2^2 = 4$

9. (a) $e^2(e^4) = e^6$

(b) $(e^3)^4 = e^{12}$

(c) $(e^3)^{-2} = e^{-6} = \frac{1}{e^6}$

(d) $\left(\frac{e^{-6}}{e^{-2}}\right)^2 = \left(\frac{e^2}{e^6}\right)^2 = \left(\frac{1}{e^4}\right)^2 = \frac{1}{e^8}$

11. $3^x = 81 \Rightarrow x = 4$

13. $6^{x-2} = 36 \Rightarrow x - 2 = 2 \Rightarrow x = 4$

15. $\left(\frac{1}{2}\right)^x = 32 \Rightarrow 2^{-x} = 32 \Rightarrow -x = 5 \Rightarrow x = -5$

17. $\left(\frac{1}{3}\right)^{x-1} = 27 \Rightarrow 3^{1-x} = 27 \Rightarrow 1 - x = 3 \Rightarrow x = -2$

19. $4^3 = (x + 2)^3 \Rightarrow 4 = x + 2 \Rightarrow x = 2$

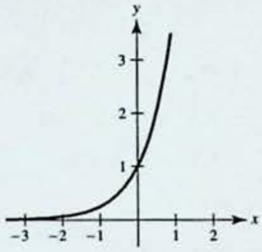
21. $x^{3/4} = 8 \Rightarrow x = 8^{4/3} = 2^4 = 16$

23. $e^x = e^{2x+1} \Rightarrow x = 2x + 1 \Rightarrow x = -1$

25. $e^{-2x} = e^5 \Rightarrow -2x = 5 \Rightarrow x = -\frac{5}{2}$

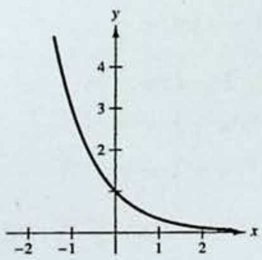
27. $y = 4^x$

x	-1	0	1	2
y	$\frac{1}{4}$	1	4	16



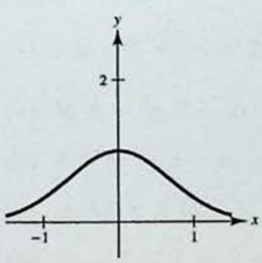
29. $y = \left(\frac{1}{3}\right)^x = 3^{-x}$

x	-2	-1	0	1	2
y	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$



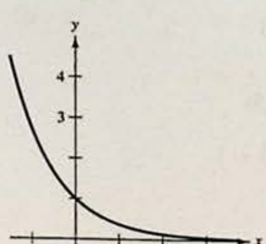
31. $f(x) = 3^{-x^2}$

x	0	± 1	± 2
y	1	$\frac{1}{3}$	0.0123



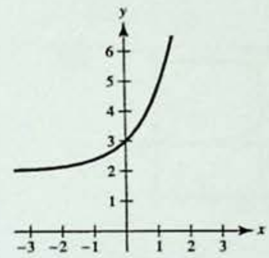
33. $y = e^{-x}$

x	-1	0	1
y	e	1	$\frac{1}{e}$



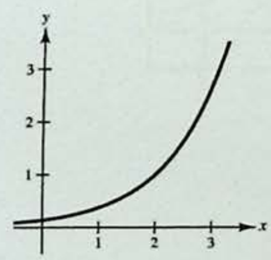
35. $y = e^x + 2$

x	-2	-1	0	1	2
y	$\frac{1}{e^2} + 2$	$\frac{1}{e} + 2$	3	$e + 2$	$e^2 + 2$



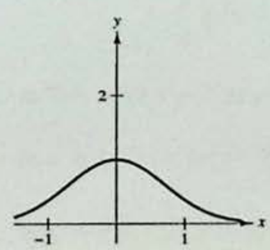
37. $h(x) = e^{x-2}$

x	0	1	2	3	4
y	e^{-2}	e^{-1}	1	e	e^2



39. $y = e^{-x^2}$

x	-1	-0.5	0	0.5	1
y	$\frac{1}{e}$	$\frac{1}{e^{1/4}}$	1	$\frac{1}{e^{1/4}}$	$\frac{1}{e}$



41. $f(x) = \dfrac{1}{3 + e^x}$

Because $e^x > 0$, $3 + e^x > 0$.

Domain: all real numbers

43. $f(x) = \sqrt{1 - 4^x}$

$1 - 4^x \geq 0 \Rightarrow 4^x \leq 1 \Rightarrow x \ln 4 \leq \ln 1 = 0$

Domain: $x \leq 0$

45. $f(x) = \sin e^{-x}$

Domain: all real numbers

47. $y = Ce^{ax}$

Graph rises from left to right

Matches (c)

48. $y = Ce^{-ax}$

Reflection in the y-axis

Matches (d)

49. $y = C(1 - e^{-ax})$

Vertical shift C units

Reflection in both the x- and y-axes

Matches (a)

50. $y = \dfrac{C}{1 + e^{-ax}}$

Matches (b)

51. $y = Ca^x$

$(0, 2): 2 = Ca^0 = C$

$(3, 54): 54 = 2a^3$

$27 = a^3$

$3 = a$

$y = 2(3^x)$

53. $e^0 = 1$

$\ln 1 = 0$

55. $\ln 4.15 = 1.4231\ldots$

$e^{1.4231\ldots} = 4.15$

57. $f(x) = \ln x + 1$

Vertical shift 1 unit upward

Matches (b)

58. $f(x) = -\ln x$

Reflection in the x-axis

Matches (d)

59. $f(x) = \ln(x - 1)$

Horizontal shift 1 unit to the right

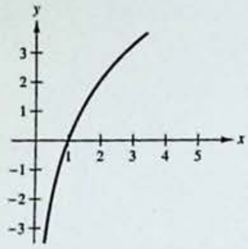
Matches (a)

60. $f(x) = -\ln(-x)$

Reflection in the y-axis and the x-axis

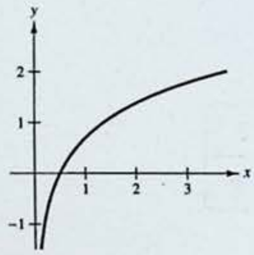
Matches (c)

61. $f(x) = 3 \ln x$



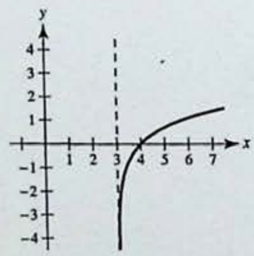
Domain: $x > 0$

63. $f(x) = \ln 2x$



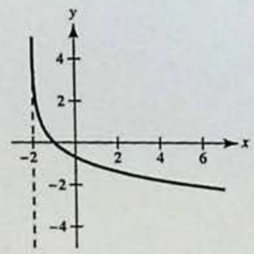
Domain: $x > 0$

65. $f(x) = \ln(x - 3)$



Domain: $x > 3$

67. $f(x) = -\ln(x + 2)$



Domain: $x > -2$

69. 8 units upward: $e^x + 8$

Reflected in x-axis: $-(e^x + 8)$

$y = -(e^x + 8) = -e^x - 8$

71. 5 units to the right: $\ln(x - 5)$

1 unit downward: $\ln(x - 5) - 1$

$y = \ln(x - 5) - 1$

www.cengage.com

CENGAGE

ISBN-13: 978-1-337-55256-1
ISBN-10: 1-337-55256-9