

Calculus

Student Text

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CALCULUS PLACEMENT TEST

I. Graph.

1.
$$y = \sin(2x)$$

2.
$$r = 6 \sin(\theta)$$

3.
$$y = In(x)$$

4.
$$f(x) = \sec(x) + 1$$

II. Evaluate.

1.
$$\sum_{i=-1}^{2} 2i^3$$

2.
$$\tan(45^{\circ}) + \sin(225^{\circ}) + \cos\left(\frac{\pi}{3}\right)$$

$$3. \qquad \lim_{x \to \infty} \ \frac{2}{x+1}$$

4.
$$\lim_{x \to 1} \frac{x^2 + 3x - 4}{x - 1}$$

5. log₂8

PLACEMENT TEST

III. Find the first four terms of the following geometric sequence with $a_1 = 3$ and $r = -\frac{1}{3}$.

IV. Solve for x.

$$1. \qquad \sqrt{x+1} < 2$$

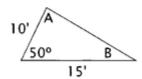
2.
$$ln(x - 1) = 3$$

3.
$$e^{2x} - 4e^x = -3$$

V. Prove the following identity.

1.
$$\sin(\theta) \cos^2(\theta) + \frac{\cos^4(\theta)}{\sin(\theta)} = \frac{\cos^2(\theta)}{\sin(\theta)}$$

VI. Solve for the unknown sides and angles in the triangle below:



VII. The decay constant of a substance is determined to be .005. How much of 100 grams will remain after 200 days? Use $Q(t) = 5e^{-kt}$ where t = time in days and Q(t) is the quantity remaining at time t.

VIII. If $f(x) = e^x$ and $g(x) = \ln(x) + 2$, then find f(g(x)).

Graph the solution(s) for x on the real number line.

- 1. 2 < x ≤ 4
- 2. |x| < 3
- 3. $|x-1| \ge 3$
- 4. $|x^3| < 8$

Draw the graph of:

- 5. y = |x| + 2
- 6. $y = -3x^3$

8.
$$y = \sqrt{x-1} - 2$$

Do any of the following graphs have a discontinuity? Is it removable? Do any have a vertical asymptote? Explain and draw a sketch.

9.
$$xy = 1$$

10.
$$y = \begin{cases} \frac{1}{x^3} & \text{for } x > 0 \\ |x| & \text{for } x \le 0 \end{cases}$$

11.
$$y = \frac{x^2 - 4}{x + 2}$$

12.
$$y = \begin{cases} \sqrt{x-2} & \text{for } x > 2 \\ |x-2| & \text{for } x \le 2 \end{cases}$$

Find the solution(s) to each problem.

1.
$$|x-1|=5$$

2.
$$|3-2x|=3$$

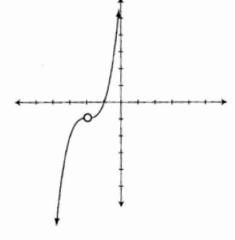
Graph the solution to each problem.

3.
$$|x-1| \le 3$$

4.
$$|2x-5|>0$$

To repair a removable discontinuity, you need to define the point that is missing. In example C in lesson 1 of the instruction manual under "Continuity", the repair would be to add the point (0,0) to the graph. What point would you add to each of the following discontinuous graphs in order to remove the discontinuity?





8.
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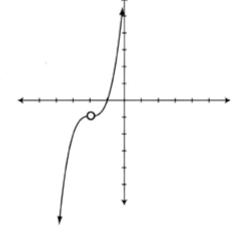
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5.



6.
$$y = \begin{cases} x-1 & \text{for } x > 2 \\ \frac{2}{x} & \text{for } x < 2 \end{cases}$$

7.
$$y = \begin{cases} \sqrt{x+2} & \text{for } x > -2 \\ 3x+6 & \text{for } x < -2 \end{cases}$$

Answer the question.

- 8. $y = \begin{cases} x \text{ for } x < 1 \\ x^2 + 1 \text{ for } x \ge 1 \end{cases}$ Does y have a discontinuity? If so, is it removable? Explain.
- 9. Explain the difference between a secant line and a tangent line.
- 10. What is the domain and range of $y = x^2 + 2$?
- 11. In lesson 1 of the instruction manual under graphing two dimensional problems, which of the six graphs shown has an asymptote? What is the equation of the asymptote? All the graphs are functions for all definable x-values. (Graph #1 is undefined when x = 0.) Excluding graph #1, which graph(s) have a range of all real numbers? Which of these graphs has a domain of x ≥ 0?

Find the solution(s) to each problem.

1.
$$|2 - x| = 3$$

2.
$$|3x + 1| = 7$$

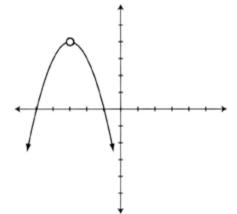
Graph the solution to each problem.

3.
$$|x-2| > 3$$

4.
$$|x^2| \le 4$$

What point(s) would you add to the following discontinuous graphs in order to remove the discontinuity?

5.



6.
$$y = \begin{cases} x^2 + 1 & \text{for } x < -1 \\ x + 3 & \text{for } x > -1 \end{cases}$$

7.
$$y = \begin{cases} x^2 \text{ for } x > 1 \\ \sqrt{x} \text{ for } 0 \le x < 1 \end{cases}$$

Answer the question.

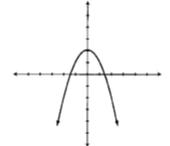
8. $y = \begin{cases} 5x + 6 \text{ for } x > 2 \\ x^2 + 1 \text{ for } x < 2 \end{cases}$ Does y have a discontinuity? If so, is it removable? Explain.

9. Define an asymptote.

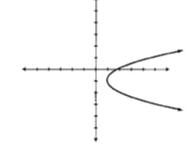
10. What is the domain and range of $y = 3 - x^2$.

Give the domain and range of the following graphs:

11.



12.



Give the name and graph for each of the following equations. Give the center, vertex, radius, extremities, and/or axis of symmetry if applicable.

1.
$$3x + y = 6$$

2.
$$y = 2x^2 + 4x - 3$$

3.
$$4x^2 + 9y^2 - 8x + 36y + 4 = 0$$

4.
$$x^2 + (y - 1)^2 = 9$$

5.
$$x = -2y$$

6.
$$x = -(y - 2)^2$$

7.
$$x^2 - 4x + y^2 - 6y + 12 = 0$$

8.
$$y = 2x^2 + 8x + 7$$

Give the name and graph for each of the following equations. Give the center, vertex, radius, extremities, and/or axis of symmetry if applicable.

1.
$$3x^2 = 9 - 3y^2$$

2.
$$x^2 + y^2 + 2x + 2y = -1$$

3.
$$x^2 - 2x + 8y = 7$$

4.
$$4y^2 + 8y + x^2 = 12$$

5.
$$2x + y = -2$$

6.
$$2x^2 + y = 3$$

7.
$$4x^2 + 9y^2 = 36$$

8.
$$x = 3y^2 - 1$$

Give the name and graph for each of the following equations. Give the center, vertex, radius, extremities, and/or axis of symmetry if applicable.

1.
$$12 - 3x^2 = 3y^2$$

2.
$$x + 3 = 2y^2$$

3.
$$2x - 5y = 10$$

4.
$$x^2 + 4y^2 + 4x = 0$$

5.
$$x^2 + 2x + y^2 - 4y = -1$$

6.
$$2x = 3y$$

7.
$$y = -3x^2 - 6x - 1$$

8.
$$9(x-2)^2 + 4(y+2)^2 = 36$$

Give the name and graph for each of the following equations. Give the center, vertex, radius, extremities, and/or axis of symmetry if applicable.

1.
$$6x - 3 = 0$$

2.
$$y^2 + 2y + 2 + x = 0$$

3.
$$x^2 + y^2 + 2x - 6y = 6$$

4.
$$2y^2 + 4y = x - 2$$

5.
$$\frac{1}{3}x + 3y - 3 = 0$$

6. $2x^2 + 4x + 3y^2 - 12y = -8$

7.
$$x^2 + y^2 + 8y = 0$$

8. $3x^2 + 3y^2 = 6x$

Graph each hyperbola showing asymptotes if applicable.

1.
$$\frac{y^2}{9} - \frac{x^2}{16} = 1$$

2.
$$xy = 7$$

Solve the following systems of equations. Prove your answer with a graph.

3.
$$16x^2 + 25y^2 = 400$$

 $x^2 + y^2 = 25$

Solve the following systems of equations. Illustrate your answer with a graph.

4.
$$y = x^2 - 2x + 2$$

 $y - 2x = -2$

5.
$$y = x^2 + 1$$

 $y - x = 1$

6. Graph
$$y < x^2 + 2$$
.

Graph each hyperbola showing asymptotes if applicable.

1.
$$xy = -2$$

2.
$$x^2 - y^2 = 16$$

3.
$$x^2 + y^2 \le 4$$

Solve the following system of equations. Illustrate your answer with a graph.

4.
$$4y = 3x$$

 $x^2 + y^2 = 25$

5.
$$xy = 6$$

 $y = -x - 5$

6.
$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

 $x^2 + y^2 = 4$

Graph each hyperbola showing asymptotes if applicable.

1.
$$y = \frac{6}{x}$$

2.
$$y^2 - x^2 = 4$$

3.
$$xy > 1$$

Solve the following systems of equations. Illustrate your answer with a graph.

4.
$$x^2 + y^2 = 4$$

 $y = x + 1$

5.
$$xy = 5$$

 $y = -x$

6.
$$\frac{x^2}{4} - y^2 = 1$$

 $y = 1$



Graph each hyperbola showing asymptotes if applicable.

1.
$$16 + x^2 = 4y^2$$

2.
$$\frac{y^2}{4} - \frac{x^2}{9} = 1$$

3.
$$x^2 - 2x + y^2 < 3$$

Solve the following system of equations. Illustrate your answer with a graph.

4.
$$y = x^2 - 2x - 3$$

 $y = -x^2 + 2x - 5$

5.
$$x^2 - y^2 = 1$$

 $x^2 + y^2 = 5$

6.
$$x^2 + y^2 = 4$$

 $4x^2 + y^2 = 4$



Solve.

1.
$$f(x) = x^3 - 5x^2 - 4x + 20$$

Find
$$f(0)$$
, $f(1)$, $f(3)$, $f(5)$.

2.
$$A(r) = \pi r^2$$

Find
$$A(0)$$
, $A(2)$, $A(3)$.

3.
$$f(x) = 4 - 2x^2 + x^4$$

Find
$$f(0)$$
, $f(1)$, $f(-1)$, $f(2)$, $f(-2)$.

4.
$$G(z) = z^2 (z^2 - 4)$$

Find
$$G(0)$$
, $G(1)$, $G(2)$, $G(-2)$.

5.
$$F(y) = 2^{y}$$

Find
$$F(0)$$
, $F(\frac{3}{2})$, $F(-1)$, $F(y - 2)$.

6.
$$f(x) = x + 2$$

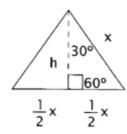
Find
$$f(x + h)$$
, $f(x + h) - f(x)$.

7. Given
$$f(x) = \frac{1}{x}$$
, show that $f(x + h) - f(x) = \frac{-h}{x^2 + xh}$.

8. Given
$$f(y) = y^2 - 2y + 6$$
, show that $f(y + h) = y^2 - 2y + 6 + 2(y - 1)h + h^2$.

Express the function by a formula and draw the graph for #9-12.

- 9. The time required for a man to travel one mile as a function of speed.
- 10. The side of a square as a function of the area.
- 11. The side of a cube as a function of the volume.
- 12. The area of a square as a function of the length of a side.
- 13. Express the area, A, of an equilateral triangle as a function of its side, x.



Calculus

PREREQUISITE: The student should have completed ALGEBRA 1, GEOMETRY, ALGEBRA 2, and PRECALCULUS or comparable courses.

- Terminology and Graphing
- Functions Review
- Trigonometry Review
- **◆** Exponent Review
- Logarithm Review
- Limits and Continuity
- ◆ Derivative
- Chain Rule
- Derivatives of Trig Functions
- Derivatives of e^x and ln x
- ◆ Implicit Differentiation
- Mean Value Theorem
- L'Hopital's Rule

- ◆ 1st and 2nd Derivative Tests
- ◆Applications: Optimization
- Applications: Business
- ◆ Applications: Physics
- ◆Applications: Related Rates
- Antidifferentiation
- Integrals
- Integration by Parts
- Differential Equation Applications
- ◆ Definite Integral
- Area under a Curve
- Physics Applications

"Math-U-See's Calculus curriculum made learning Calculus easy, and imparted an in-depth understanding of how it works and how to apply it practically. Math-U-See's distinct step-by-step learning that is found in all of their previous curricula is continued in this course as well. The videos were very informative and laid a solid foundation for each lesson. I am currently in a community college Calculus class, and have gotten straight-A's so far – thanks to this curriculum."

-Daniel Tekunoff, AZ

The Math-U-See strategy at this level involves two components: video and written instruction with examples for the teacher and student, and textbooks to provide practice, long term review, and real life applications.

Watch the DVD until you understand the new topic, read the instruction manual and study the examples, and work through the student text until the math concept has been internalized. Then take the test to demonstrate that this topic has been mastered before moving to the next lesson.

For success with Calculus, we recommend these materials:



CALCULUS INSTRUCTION PACK



CALCULUS STUDENT KIT

mathusee.com

