SAXON° ALGEBRA 2



SAXON° ALGEBRA 2

Student Edition



www.SaxonPublishers.com 800-531-5015

ISBN 13: 978-1-6027-7303-5

ISBN 10:

1-6027-7303-3

© 2009 Saxon*, an imprint of HMH Supplemental Publishers Inc.

All rights reserved. No part of this material protected by this copyright may be reproduced or utilized in any form or by any means, in whole or in part, without permission in writing from the copyright owner. Requests for permission should be mailed to: Paralegal Department, 6277 Sea Harbor Drive, Orlando, FL 32887.

Saxon® is a registered trademark of HMH Supplemental Publishers Inc.

Printed in the United States of America

If you have received these materials as examination copies free of charge, HMH Supplemental Publishers Inc. retains title to the materials and they may not be resold. Resale of examination copies is strictly prohibited and is illegal.

Possession of this publication in print format does not entitle users to convert this publication, or any portion of it, into electronic format.

10 11 12 1421 18 17 16 4500606458

Table of Contents

Section 1: Lessons 1-10, Investigation 1

	LESSON		PAGE
	1	Using Properties of Real Numbers	2
	2	Evaluating Expressions and Combining Like Terms	8
	3	Using Rules of Exponents	13
	LAB 1	Graphing Calculator: Graphing a Function and Building a Table	19
	4	Identifying Functions and Using Function Notation	21
	LAB 2	Graphing Calculator: Storing and Recalling Data in a Matrix	27
	5	Using Matrices to Organize Data and to Solve Problems	29
	6	Finding Percent of Change	36
	7	Solving Linear Equations Exploration: Solving Equations Using Algebra Tiles	42
	8	Finding Direct Variation	48
V in	9	Multiplying Matrices	54
707	10	Solving and Graphing Inequalities	61
	INV 1	Investigation: Logic and Truth Tables	69

DISTRIBUTED STRANDS

Number Sense and Foundations of Algebra

Linear Functions

Matrices

Polynomials and Polynomial Functions

Rational and Radical Functions

Linear Systems

Probability and Statistics

Quadratic Functions

Trigonometry

Exponential and Logarithmic Functions

Conic Sections

Sequences, Series, and Logic

Section 2: Lessons 11-20, Investigation 2

LESSON		PAGE
11	Understanding Polynomials	72
12	Solving Inverse Variation Problems	77
LAB 3	Graphing Calculator: Calculating Points on a Graph	84
13	Graphing Linear Equations I	86
14	Finding Determinants	93
15	Solving Systems of Equations by Graphing	100
16	Using Cramer's Rule	107
LAB 4	Graphing Calculator: Changing the Line and Window of a Graph	114
17	Solving Equations and Inequalities with Absolute Value Exploration: Transforming $f(x) = x $	116
18	Calculating with Units of Measure	124
19	Multiplying Polynomials	129
20	Performing Operations with Functions	136
INV 2	Investigation: Solving Parametric Equations	143

Section 3: Lessons 21-30, Investigation 3

	LESSON		PAGE
	21	Solving Systems of Equations Using the Substitution Method	146
	LAB 5	Graphing Calculator: Storing and Plotting a List of Data	153
	22	Analyzing Continuous, Discontinuous, and Discrete Functions	155
100	23	Factoring Polynomials	163
	24	Solving Systems of Equations Using the Elimination Method	170
100	LAB 6	Graphing Calculator: Calculating 1- and 2-Variable Statistical Data	178
	25	Finding Measures of Central Tendency and Dispersion	180
	26	Writing the Equation of a Line	187
	27	Connecting the Parabola with the Quadratic Function	194
	28	Simplifying Rational Expressions	201
	29	Solving Systems of Equations in Three Variables	207
	30	Applying Transformations to the Parabola and Determining the Minimum or Maximum	215
	INV 3	Investigation: Graphing Three Linear Equations in Three Variables	222

DISTRIBUTED STRANDS

Number Sense and Foundations of Algebra

Linear Functions

Matrices

Polynomials and Polynomial Functions

Rational and Radical Functions

Linear Systems

Probability and Statistics

Quadratic Functions

Trigonometry

Exponential and Logarithmic Functions

Conic Sections

Sequences, Series, and Logic

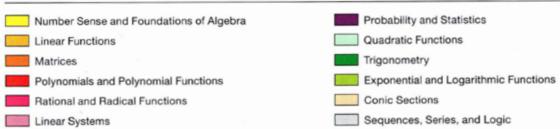
Section 4: Lessons 31-40, Investigation 4

	LESSON		PAGE
	31	Multiplying and Dividing Rational Expressions	226
	32	Solving Linear Systems with Matrix Inverses Exploration: Exploring Matrix Inverses	233
14/4	33	Applying Counting Principles	240
	34	Graphing Linear Equations II	246
	35	Solving Quadratic Equations I	253
	36	Using Parallel and Perpendicular Lines	259
	37	Adding and Subtracting Rational Expressions	266
	38	Dividing Polynomials Using Long Division	273
	39	Graphing Linear Inequalities in Two Variables	279
	40	Simplifying Radical Expressions	286
	INV 4	Investigation: Understanding Cryptography	293

Section 5: Lessons 41-50, Investigation 5

	LESSON		PAGE
ÇK.	41	Using the Pythagorean Theorem and the Distance Formula Exploration: Visualizing the Pythagorean Theorem	296
	LAB 7	Graphing Calculator: Calculating Permutations and Combinations	303
	42	Finding Permutations and Combinations Exploration: Pascal's Triangle and Combinations	304
	43	Solving Systems of Linear Inequalities	312
	44	Rationalizing Denominators	318
	LAB8	Graphing Calculator: Applying Linear and Median Regression	323
	45	Finding the Line of Best Fit Exploration: Collecting and Analyzing Data	325
	46	Finding Trigonometric Functions and their Reciprocals	331
P	47	Graphing Exponential Functions	337
	48	Understanding Complex Fractions	343
	49	Using the Binomial Theorem	348
	50	Finding Inverses of Relations and Functions Exploration: Graphing a Function and its Inverse	355
566	INV 5	Investigation: Finding the Binonomial Distribution	361

DISTRIBUTED STRANDS



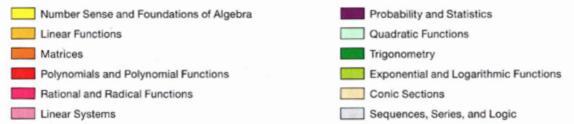
Section 6: Lessons 51-60, Investigation 6

	LESSON		PAGE
	51	Using Synthetic Division	364
	LAB 9	Graphing Calculator: Using the Trigonometry Keys and Adjusting to a Trigonometric Window	370
	52	Using Two Special Right Triangles	372
	53	Performing Compositions of Functions	378
	54	Using Linear Programming	384
	55	Finding Probability	391
	56	Finding Angles of Rotation	399
	57	Finding Exponential Growth and Decay	406
	58	Completing the Square Exploration: Modeling Completing the Square	413
	59	Using Fractional Exponents	420
40	60	Distinguishing Between Mutually Exclusive and Independent Events	427
	INV 6	Investigation: Deriving the Quadratic Formula	433

Section 7: Lessons 61-70, Investigation 7

	LESSON		PAGE
	61	Understanding Advanced Factoring	436
	62	Using Complex Numbers	442
	63	Understanding the Unit Circle and Radian Measures Exploration: Exploring the Unit Circle	447
	LAB 10	Graphing Calculator: Using the Log Keys	455
	64	Using Logarithms	457
	65	Using the Quadratic Formula	462
	66	Solving Polynomial Equations	469
	67	Finding Inverse Trigonometric Functions	476
7 40	68	Finding Conditional Probability	483
	69	Simplifying Complex Expressions	489
plants.	70	Solving Radical Equations	495
260	INV 7	Investigation: Collecting Data	502

DISTRIBUTED STRANDS



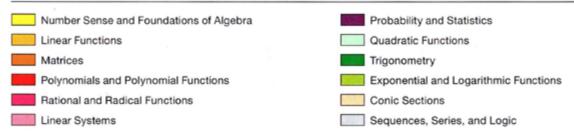
Section 8: Lessons 71-80, Investigation 8

	LESSON		PAGE
	71	Using the Law of Sines	506
787	72	Using the Properties of Logarithms Exploration: Discovering the Properties of Logarithms	512
	LAB 11	Graphing Calculator: Calculating Confidence Intervals	519
	73	Using Sampling	521
	74	Finding the Discriminant	528
	75	Graphing Radical Functions	534
	76	Finding Polynomial Roots I	540
	77	Using the Law of Cosines	546
5.32	78	Solving Quadratic Equations II	552
	79	Understanding Piecewise Functions	558
-10	LAB 12	Graphing Calculator: Calculating Normal Distribution Areas and Z-Scores	563
	80	Finding the Normal Distribution	565
	INV 8	Investigation: Finding the Area Under a Curve	572

Section 9: Lessons 81-90, Investigation 9

_	LESSON		PAGE
	81	Using Natural Logarithms	574
	82	Graphing Sine and Cosine Functions	580
	83	Writing Quadratic Equations from Roots	586
	84	Solving Rational Equations Exploration: Exploring Rational Functions	592
	85	Finding Polynomial Roots II	598
	86	Translating Sine and Cosine Functions	606
	87	Evaluating Logarithmic Expressions	612
	88	Solving Abstract Equations	617
	89	Solving Quadratic Inequalities	623
M.V.	90	Graphing the Tangent Function	630
	INV 9	Investigation: Understanding Step Functions	635

DISTRIBUTED STRANDS



Section 10: Lessons 91-100, Investigation 10 **LESSON** PAGE LAB 13 Graphing Calculator: Graphing in Circles and Polar Equations 638 Making Graphs and Solving Equations of Circles 91 640 Finding Arithmetic Sequences 92 646 **Exploration:** Exploring Sequences 93 Solving Exponential Equations and Inequalities 652 94 Solving Rational Inequalities 658 95 Factoring Higher-Order Polynomials 665 96 Using Polar Coordinates 671 97 Finding Geometric Sequences 678 Making Graphs and Using Equations of Ellipses 98 684 99 Using Vectors 690 100 Graphing Rational Functions I 696 **INV 10 Investigation:** Graphing Polar Models 703

Section 11: Lessons 101-110, Investigation 11

	LESSON		PAGE
177	101	Making Graphs of Polynomial Functions	706
	102	Solving Logarithmic Equations and Inequalities	714
1000	103	Graphing Reciprocal Trigonometric Functions	719
	104	Finding Transformations	725
	105	Finding Arithmetic Series	732
	106	Using the Fundamental Theorem of Algebra Exploration: Making Conjectures About Roots of Polynomial Functions	738
	107	Graphing Rational Functions II	745
0.4	108	Using Fundamental Trigonometric Identities	752
	109	Making Graphs and Using Equations of Hyperbolas	757
	110	Graphing Logarithmic Functions	764
	INV 11	Investigation: Using De Moivre's Theorem	770

DISTRIBUTED STRANDS



Section 12: Lessons 111-120, Investigation 12

LESSON		PAGE
111	Transforming Polynomial Functions	774
112	Using Sum and Difference Identities	780
113	Using Geometric Series Exploration: Exploring an Infinite Geometric Series	786
114	Identifying Conic Sections	793
115	Finding Double-Angle and Half-Angle Identities	798
LAB 14	Graphing Calculator: Determining Regression Models	804
116	Finding Best Fit Models	806
117	Solving Systems of Nonlinear Equations	813
118	Recognizing Misleading Data	819
119	Solving Trigonometric Equations	825
INV 12	Investigation: Using Mathematical Induction	831

Appendix LESSON		834
		PAGE
1	Changes in Measure	834
2	Computer Spreadsheets	838
3	Precision and Accuracy	842
4	Predictions	846
5	Scale Factor	850
6	Regions and Solids	854
7	Apply Scientific Notation	858

Skills	Bank	862
LESSO	N	PAGE
1	Estimation	862
2	Mental Math	863
3	Exponents	864
4	Operations with Decimals	865
5	Compare and Order Rational Numbers	866
6	Operations with Fractions	867
7	Negative Numbers/Operations with Integers	868
8	Ratios, Proportions, Percents	869
9	Time, Rate, Distance	870
10	Coordinate Plane/Ordered Pairs	871
11	Plane Figures and Coordinate Geometry	872
12	Parallel Lines and Transversals	873
13	Measure Angles	874
14	Angle Relationships	875
15	Properties of Polygons	876
16	Geometric Formulas	877
17	Area of Polygons, Circles, and Composite Figures	878

ESSO	N	PAGE
18	Angle Relationships in Circles and Polygons	879
19	Views of Solid Figures	880
20	Geometric Patterns and Tessellations	881
21	Stem and Leaf Plots	882
22	Statistical Graphs	883
23	Proofs	884
24	Venn Diagrams	885

perties and Formulas	886
	PAGE
Properties	886
Formulas	890
Symbols	891
Metric Measures	892
Customary Measures	892

Glossary	ALC: N	Health I			894
	-70.74	III DON'T BE	2012	CI-S-SHIPS	
Index	LANG	Marie 1		Show the	957

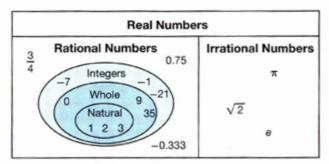
Using Properties of Real Numbers

Warm Up

Start off each lesson by practicing prerequisite skills and math vocabulary that will make you more successful with today's new concept.

- 1. Vocabulary A number whose value is less than 0 is called a number.
- 2. Add 12 + (-12).
- 3. Multiply $\frac{2}{3} \cdot \frac{3}{2}$.
- **4.** Subtract 10 9.85.

New Concepts A set is a collection of objects. If all of the objects in one set are also members of a second set, then the first set is a subset of the second. The real numbers are a set of numbers consisting of several subsets of numbers. The real numbers consist of the rational numbers and the irrational numbers. The rational numbers include the natural numbers {1, 2, 3, ...}, the whole numbers $\{0, 1, 2, 3, ...\}$, and the integers $\{..., -3, -2, -1, 0, 1, 2, 3, ...\}$.



Rational numbers can be written as quotients of integers. Rational numbers can also be written as decimals that terminate or repeat. Irrational numbers cannot be written in any of these ways. Pi and many roots of real numbers are common examples of irrational numbers.

Caution

The negative integers are not considered whole numbers even though they do not contain fractions.

Example 1 **Classifying Real Numbers**

Identify the subsets of real numbers of which each number is a member.

$$(a.)$$
 -5

SOLUTION -5 is a real number, a rational number, and an integer.

SOLUTION 25 is a real number, a rational number, an integer, a whole number, and a natural number.

$$\left(\mathbf{c.} \right) - \sqrt{5}$$

SOLUTION $-\sqrt{5}$ is a real number and an irrational number.

There are a number of properties that can be used when adding and multiplying real numbers.

Properties of Addition and Multiplication

Let a, b, and c be real numbers.

Property Addition

a + b is a real number

Multiplication

Closure

ti | O lo ti letti lit

ab is a real number

Commutative

$$a+b=b+a$$

$$ab = ba$$

Associative

$$(a+b) + c = a + (b+c)$$

$$(ab)c = a(bc)$$

The following property involves both addition and multiplication.

Distributive

$$a(b+c) = ab + ac$$

Example 2 Identifying Properties of Real Numbers

Identify which property of real numbers is being demonstrated.

(a.)
$$3 \cdot 8 = 8 \cdot 3 = 24$$

SOLUTION Commutative Property of Multiplication

b.
$$5(7+11) = 5 \cdot 7 + 5 \cdot 11 = 35 + 55 = 90$$

SOLUTION Distributive Property

$$(12 + 20) + 30 = 12 + (20 + 30) = 12 + 50 = 62$$

SOLUTION Associative Property of Addition

Example 3 Using the Properties of Real Numbers

Simplify each expression. Identify which property you used for each step.

$$a. 12 + 4 + 18 + 56$$

SOLUTION

$$12 + 4 + 18 + 56$$

$$= 12 + 18 + 4 + 56$$

Commutative Property of Addition

$$=(12+18)+(4+56)$$

Associative Property of Addition

$$= 30 + 60$$

Add.

$$= 90$$

Add.

SOLUTION

$$=5(20+3)$$

$$= (5 \cdot 20) + (5 \cdot 3)$$

Distributive Property

$$= 100 + 15$$

Multiply.

$$= 115$$

Add.

Hint

The Distributive Property makes multiplying easier by breaking down a number into numbers where mental math can be used.

Each day brings you a New Concept where a

new topic is introduced and explained through

thorough Examples
— using a variety of
methods and real-world

You will be reviewing and building on this concept throughout

the year to gain a solid understanding and

ensure mastery on the

applications.

test.

More Properties of Addition and Multiplication

	role r toperties of reduction and re-	untiplication
Property	Addition	Multiplication
Identity	a + 0 = a, $0 + a = a$	$a \cdot 1 = a, 1 \cdot a = a$
Inverse	a + (-a) = 0	$a \cdot \frac{1}{a} = 1, a \neq 0$

Math Language

The **additive inverse** of a number is sometimes called the opposite of the number.

In some lessons,
Explorations allow you to go into more depth with the mathematics by investigating math concepts with manipulatives, through patterns, and in a variety of other ways.

The Lesson Practice lets you check to see if you understand today's new concept.

The italic numbers refer to the Example in this lesson in which the major concept of that particular problem is introduced. You can refer to lesson examples if you need additional help.

Example 4 Finding Inverses of Real Numbers

[a.] Find the additive inverse of -7a.

SOLUTION The additive inverse of -7a is 7a since -7a + 7a = 0.

b. Find the multiplicative inverse of $\frac{5n}{12p}$.

SOLUTION The multiplicative inverse is the reciprocal of $\frac{5n}{12p}$ which is $\frac{12p}{5n}$.

$$\frac{5n}{12p} \cdot \frac{12p}{5n} = 1$$

Example 5 Application: Finance

DVDs are on sale at Tech World for \$11.95 each including tax. Use the Distributive Property to mentally calculate the total cost of buying 5 DVDs.

SOLUTION Think of \$11.95 as \$12.00 - \$0.05.

Write an expression for the cost of 5 DVDs.

$$5(12 - 0.05)$$

$$= 5(12) - 5(0.05)$$
 Use the Distributive Property.

$$= 60 - 0.25$$
 Multiply.
= 59.75 Subtract.

The total cost of 5 DVDs is \$59.75.

Lesson Practice

- **a.** Identify the subsets of real numbers of which $\sqrt{3}$ is a member.
- **b.** Identify the subsets of real numbers of which $-\frac{2}{3}$ is a member.
- **c.** Identify the subsets of real numbers of which 1 is a member. (Ex I)
- **d.** Identify which property of real numbers is being demonstrated.

$$-2(5+9) = -2 \cdot 5 + (-2) \cdot 9 = -10 - 18 = -28$$

e. Identify which property of real numbers is being demonstrated.

$$(12 \cdot 6) \cdot 2 = 12 \cdot (6 \cdot 2) = 12 \cdot 12 = 144$$

f. Identify which property of real numbers is being demonstrated. $\frac{f}{(E \times 2)}$

$$3 + 21 = 21 + 3 = 24$$

g. Simplify the expression (43 + 21) + 9. Identify which property you used (Ex3) for each step.

- **h.** Simplify the expression $(4 \cdot 16) \cdot 4$. Identify which property you used for each step.
- i. Find the additive inverse of $\frac{3q}{2}$.
 - **j.** Find the multiplicative inverse of -4.
- **k.** Lucinda drove four hours from her house to a family reunion. Her average speed was 55 mph. Use the Distributive Property to mentally calculate the distance between Lucinda's house and the reunion.

Practice Distributed and Integrated

Simplify each expression.

1.
$$-3 - 6 + 1$$

$$2. -4 + 6 - 8 =$$

Simplify each expression using the properties of real numbers.

*3.
$$-2[(5-7-2)-(-2-7)-2]$$
 *4. $4[3-(-2)]+5(-2+1)$

*4.
$$4[3-(-2)]+5(-2+1)$$

The italic numbers refer to the lesson(s) in which the major concept of that particular problem is introduced. You can refer to the examples or practice in that lesson, if you need additional help.

Simplify each expression.

6.
$$39.75 + 49.2$$

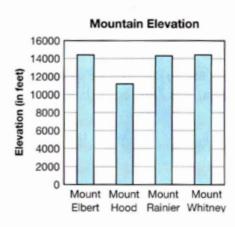
Order the given numbers from least to greatest.

8.
$$\frac{3}{5}$$
, $\frac{4}{7}$, $\frac{1}{2}$

Find the distance traveled given the rate and time below.

- **9.** rate = 40 miles per hour, time = $3\frac{1}{2}$ hours
- 10. rate = 55 miles per hour, time = 0.5 hours
- *11. Write Explain the steps you would use to simplify -2(28 19) + 6. Then find
 - *12. Justify Simplify 2 · 3 · 6. Justify your answer by identifying which property you
 - 13. (Geography) The following graph represents the elevations of mountains located in the United States.

Which mountain has the lowest elevation?



- **14.** Sports A football field is 120 yards long and 160 feet wide. What is the area in square yards?
- *15. Verify Simplify the expression (6 5) 2 using only one property of real numbers.
 (1) Identify the property that you used. Verify your result using two properties of real numbers.
- *16. Multiple Choice Identify the subset of real numbers of which 7 is not a member.

A irrational number

B integer

C whole number

D real number

- 17. Multi-Step Suzy can read 12 pages in four minutes. Jonas can read 9 pages in three minutes.
 - a. Find the number of pages Suzy and Jonas can read per minute.
 - b. Compare the rates.
- *18. Multiple Choice Identify which property or properties of real numbers are being demonstrated.

$$27 \cdot 3 = (20 \cdot 3) + (7 \cdot 3) = 60 + 21 = 81$$

- A Commutative Property of Addition
- B Associative Property of Addition
- C Both A and B
- D Distributive Property
- 19. Sports The specified circumference of an official professional baseball is from 9 to 9.25 inches.
 - a. Find the approximate radius of the smallest official baseball.
 - b. Find the approximate surface area of the smallest official baseball.
- *20. Statistics The table below shows the number of homeruns hit by a softball player over a three-year period. Use properties of real numbers to mentally calculate the average annual homeruns that she hit during this period.

	Year 1	Year 2	Year 3
Homeruns	6	13	14

- *21. Interest Rates The average credit card annual interest rate is around 19%. Use the Distributive Property to mentally calculate the amount of interest paid annually on a \$500 balance.
- The mixed set of Practice is just like the mixed format of your state test. You'll be practicing for

the "big" test every day!

In the Practice, you

will review today's new concept as well as math

you learned in earlier lessons. By practicing

problems from many lessons every day, you

will begin to see how math concepts relate and

connect to each other

and to the real world.

in a variety of ways over several lessons, you will

have "time to learn" the concept and will have

opportunities to show

that you understand.

Also, because you practice the same topic

- 22. Biology Some large species of bamboo plants can grow at a rate of 1 meter per day.
 - a. If a 2-meter tall bamboo plant grows at this rate, how tall will it be after 2 weeks?
 - b. How long will it take for the bamboo plant to reach a height of 50 meters?

6

*23. Geometry The base of a triangle equals $\frac{1}{m}$ and the height equals m. The area of the triangle can be calculated as shown below. Which properties of real numbers are demonstrated?

$$A = \frac{1}{2}(bh) = \frac{1}{2}(\frac{1}{m} \cdot m) = \frac{1}{2} \cdot 1 = \frac{1}{2}$$

- **24.** Carlos measures an angle x. He then finds that the complement of the angle is 35° .
 - a. What is the measure of angle x?
 - **b.** What is the supplement of angle x?
- *25. Error Analysis Explain the error in the statement that follows and correct the statement. "-7 is a real number, a rational number, and a whole number."
 - **26. Estimate** The chart below shows the total amount of hybrid vehicles sold by a manufacturer over a 4-year time span. Round to the nearest ten-thousand to estimate:
 - a. the total number of hybrid vehicles sold from 2003 to 2006.
 - b. the sales increase from 2003 to 2006.

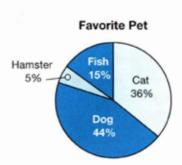
Year	2003	2004	2005	2006
Cars Sold	47,525	83,153	209,711	246,642

- *27. Graphing Calculator Use the Distributive Property to mentally calculate 12(18). Check using a graphing calculator.
 - 28. Error Analysis Two students calculated the total cost for 3 CDs. Each CD cost \$14.95. Which student has the correct answer? Explain the other student's error.

Student A	Student B
3(15 - 0.05)	3(15-0.05)
3(15) - (0.05)	3(15) - 3(0.05)
45 - 0.05	45 - 0.15
44.95	44.85

- 29. A cube has a side length of 5 centimeters.
 - a. Find the surface area of the cube.
 - b. Find the volume of the cube.
- 30. Analyze John asked 100 people to choose their favorite pet.

Use the chart to determine how many people chose dogs as their favorite pet.



The starred problems usually cover challenging or recently presented

content. Because of that, it is suggested that these exercises be worked first,

in case you might want

Evaluating Expressions and Combining Like Terms

Warm Up

- 1. Vocabulary In the term 3xy, 3 is the _
- **2.** True or False: 23 + 2(4) = 5(6) + 7.
- 3. Simplify: $8 + 12 \div 4 5$.

New Concepts An algebraic expression can contain letters that represent unspecified numbers. These letters are called variables. The value of the algebraic expression x + 4 depends on the number you use as a replacement for x. If you replace x with -32, the expression will have a value of -28.

$$x + 4$$

$$(-32) + 4 = -28$$

When you replace the variables in an expression with selected numbers and simplify using the order of operations, you have evaluated the expression.

Order of Operations

- Parentheses and grouping symbols
- Exponents
- Multiply and divide from left to right
- 4. Add and subtract from left to right

Example 1 Evaluating Expressions with Exponents

Evaluate each expression if x = -2 and y = -4.

(a.) $x^2y - y$

SOLUTION

$$x^2y - y$$

 $= (-2)^2(-4) - (-4)$ Replace x with -2 and y with -4 .
 $= (+4)(-4) - (-4)$ Perform operations with exponents.
 $= -16 - (-4)$ Multiply.
 $= -16 + 4 = -12$ Subtract.

b.
$$2xy + 3y^2$$

SOLUTION

$$2xy + 3y^2$$

= $2(-2)(-4) + 3(-4)^2$ Replace x with -2 and y with -4 .
= $2(-2)(-4) + 3(16)$ Perform operations with exponents.
= $16 + 48 = 64$ Multiply and add.

Hint

opposite.

Subtracting a number is

the same as adding its

Example 2 Evaluating Expressions with Parentheses

Evaluate each expression if a = -2 and b = 4.

$$a \cdot a(-b-a) - ab$$

SOLUTION

$$a(-b-a)-ab$$

$$=-2[-(4)-(-2)]-(-2)(4)$$

4)

Replace a with -2 and b with 4.

$$=-2(-2)-(-2)(4)$$

Perform operations inside parentheses.

$$=4-(-8)$$

Multiply.

$$= 4 + 8$$

Subtract.

$$= 12$$

b.
$$ab - a + \frac{a+b}{2}$$

SOLUTION

$$ab - a + \frac{a+b}{2}$$

$$=(-2)(4)-(-2)+\frac{(-2)+4}{2}$$

Replace a with −2 and b with 4.

$$=(-2)(4)-(-2)+\frac{2}{2}$$

Perform operations inside parentheses.

$$=(-8)-(-2)+1$$

Divide.

$$= -8 + 2 + 1$$

Add and subtract.

$$= -5$$

Example 3 Using a Calculator to Evaluate Expressions

Use a calculator to evaluate $3mp + 2p^2$ if m = 6 and p = -3.

SOLUTION Replace m with 6 and p with -3.

$$3(6)(-3) + 2(-3)^2$$

Enter this expression into the calculator as shown.

Press ENTER to find the answer is -36.

Caution

When using your calculator to square negative numbers you need to use parentheses. $(-3)^2 = 9$, but $-3^2 = -9$.

The terms of an algebraic expression are separated by addition and subtraction symbols. Like terms have the same variable raised to the same power. Constant terms are like terms that always have the same value.

Like terms
$$5x^2 + 4 - 8xy + 3x^2 - 1$$
Constant terms

Add like terms by adding the coefficients of the terms, as shown in the following examples.

Example 4 Simplifying Expressions

Simplify the expression by adding like terms.

$$3xy - 2x + 4 - 6yx + 3x$$

SOLUTION Rearrange the expression so that the like terms are next to each other. Then add like terms.

$$3xy - 2x + 4 - 6yx + 3x$$

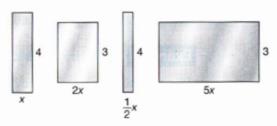
= $3xy - 6yx - 2x + 3x + 4$
= $-3xy + x + 4$

Hint

The order of the variables that a term is composed of doesn't change its value (xyz equals zyx).

Example 5 Application: Metalworking

A metal worker is using the four sheets of metal shown below with their dimensions. What is the total area of the four sheets?



SOLUTION

First find the individual areas.

$$4 \cdot x = 4x$$
 square units
 $3 \cdot 2x = 6x$ square units
 $4 \cdot \frac{1}{2}x = 2x$ square units
 $3 \cdot 5x = 15x$ square units

Now add to find the total area.

$$4x + 6x + 2x + 15x = 27x$$
 square units

Lesson Practice

a. Evaluate
$$2x - 5x^2y$$
 if $x = 3$ and $y = -2$.

b. Evaluate
$$4xy^2 + x^2$$
 if $x = -3$ and $y = -1$.

c. Evaluate
$$\frac{2(a+b)+4}{4} - a$$
 if $a = -4$ and $b = 6$.

d. Evaluate
$$-3ab - b(4 - a)$$
 if $a = -2$ and $b = 7$.

e. Use a calculator to evaluate
$$p(-p+1) - m$$
 if $m = 5$ and $p = -3$.

f. Simplify the expression by adding like terms.
$$(Ex 4)$$

$$-xy + 9 - 7x - 9xy + 7$$

g. At the local drugstore, notebooks cost
$$7x$$
 dollars, pens cost $2x$ dollars, folders cost $3x$ dollars, and highlighters cost $4x$ dollars. Trent bought 1 notebook, 2 pens, 3 folders, and 1 highlighter. What was the total cost?

Practice Distributed and Integrated

Find the volume of the rectangular prisms with the given dimensions.

1.
$$l = 2$$
 inches, $w = 4$ inches, $h = 9$ inches

2.
$$l = 0.5$$
 cm, $w = 1.2$ cm, $h = 6$ cm

- 3. Golf Tiger Woods obtained an average score of 68 during the four rounds of golf at the PGA Championship in 2007. Use the Distributive Property to mentally calculate his total score for the four rounds of golf.
- *4. (Amusement Parks) An amusement park in Atlanta, Georgia offers a child's ticket for \$29.99 and an adult ticket for \$39.99.
 - a. Round individual ticket prices to the nearest dollar and write an expression for the total ticket cost for a family with A adults and C children.
 - b. Which would pay more, a family with one adult and four children, or a family with three adults and one child? Explain.

Evaluate each expression for the given values of x and y.

*5.
$$x - xy^2 - xy$$
 if $x = -2$ and $y = -3$

*5.
$$x - xy^2 - xy$$
 if $x = -2$ and $y = -3$ *6. $(x - y) - x(-y)$ if $x = -5$ and $y = 3$

*7.
$$x^2(x - xy)$$
 if $x = -2$ and $y = 3$

*7.
$$x^2(x-xy)$$
 if $x=-2$ and $y=3$
*8. $x^2-y(x-y)$ if $x=-\frac{1}{2}$ and $y=\frac{1}{4}$

*9.
$$xy(1-y)$$
 if $x = \frac{1}{5}$ and $y = -10$

*9.
$$xy(1-y)$$
 if $x=\frac{1}{5}$ and $y=-10$ *10. $xy-(x^2-y)$ if $x=-\frac{1}{3}$ and $y=\frac{1}{2}$

Identify the subsets of real numbers of which each number is a member.

11.
$$\sqrt{8}$$

13.
$$\frac{2}{3}$$

*14. Budgets Van budgets \$12 a day for groceries for weekdays and \$15 a day for weekends. Write an expression in simplest form to show his grocery budget for w weeks.

*15. Multi-Step For the expression 2qr(3+r)

- **a.** evaluate if q = 2 and r = -1.
- **b.** explain how the result would be affected if q was doubled.
- c. show this is true by evaluating the expression if q = 4 and r = -1.

*16. Sports A football is kicked from ground level. The height h in feet of the ball after t seconds can be modeled by the expression $-16t^2 + 48t$. Find the height of the ball after 2 seconds.

Measurement Find the measure of the complementary angle given the measures of the angles below.

*21. Multiple Choice Evaluate $4x^2 + 7y$ if x = 3 and y = -2.

A 10

B 41

C 50

D 22

22. Error Analysis Two students multiplied $-13 \cdot 18$ using the Distributive Property. Which student is correct? Find and explain the other student's error.

Student A	Student B
$-13 \cdot 18 = -13(20 - 2)$	$-13 \cdot 18 = -13(20 - 2)$
=-13(20)+13(2)	=-13(20)-13(2)
= -260 + 26	= -260 - 26
= -234	= -286

*23. Geometry A rectangle has length c and width 2d. Another rectangle has length 4 and width (5-c). Write a simplified expression of their combined area.

*24. Verify Is $a^2 + b^2 = (a + b)^2$? Support your answer with an example.

*25. Consumer A local sandwich shop has \$6 lunches, and you receive a \$5 discount on the tenth lunch. Calculate your total cost for ten lunches at this shop.

26. Write Does zero have a multiplicative inverse? Explain your answer.

*27. Graphing Calculator Use a calculator to evaluate the expression $fg^2 - (2f - g^2)$ if f = 1 and g = -2. Then, verify the answer by simplifying and solving.

Simplify by combining like terms.

*28. 5ab + 7a - 3ab + 4b *29. $y - 9x^2y + 4 + 3x^2y + 12$ *30. 2(x + 3) - x

Using Rules of Exponents

Warm Up

Reading Math In the expression 54, 5

4th power."

is the base and 4 is the exponent. The expression is read "5 raised to the

- 1. Vocabulary In the expression 3^2 , 2 is the
- 2. True or False: $5^4 = 5 + 5 + 5 + 5$.
- 3. Evaluate $n \cdot n \cdot n \cdot n$ for n = -3.

New Concepts In an expression such as x^n the exponent tells the number of times that the base is a factor. So 23 is defined as 2 times 2 times 2. Two to the negative third power is defined as 1 over 2 to the third power.

$$2^{-3} = \frac{1}{2^3}$$

The formal definition of negative exponents is as follows:

Definition of x^{-n}

If n is any real number and x is any real number that is not zero,

$$x^{-n} = \frac{1}{x^n}.$$

This definition says that when an exponential expression is written in reciprocal form, the sign of the exponent must be changed. If the exponent is negative, it is positive in reciprocal form; and if it is positive, it is negative in reciprocal form.

Simplifying Negative Exponents Example 1

Simplify.

(a.)
$$\frac{1}{3^{-2}}$$

SOLUTION

a.
$$\frac{1}{3^{-2}} = 3^2 = 9$$

b.
$$3^{-3} = \frac{1}{3^3} = \frac{1}{27}$$

$$(c.)$$
 -3^{-2}

d.
$$(-3)^{-2}$$

SOLUTION

c.
$$-3^{-2} = -\frac{1}{3^2} = -\frac{1}{9}$$

d.
$$(-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{9}$$

$$(e.)$$
 $-(-3)^{-3}$

SOLUTION

e.
$$-(-3)^{-3} = -\frac{1}{(-3)^3} = -\frac{1}{-27} = -\left(-\frac{1}{27}\right) = \frac{1}{27}$$

Math Reasoning

Connect Apply the definition of negative exponents to explain why the product theorem states that $x \neq 0$.

Math Reasoning

Analyze In Example 2 the definition of x^{-n} is used to simplify each expression. What mathematical operation would have the same result? Explain.

Two rules for exponents are the product rule and the power rule. These rules simplify operations with bases.

The product of powers whose bases are the same can be found by writing each power as repeated multiplication. $x^2 \cdot x^3$ means $x \cdot x$ times $x \cdot x \cdot x$ which equals x^5 . The exponents may be added to obtain the same result: $x^2 \cdot x^3 = x^{2+3} = x^5$. This demonstrates the product rule for exponents.

Product Rule for Exponents

If m, n, and x are real numbers and $x \neq 0$,

$$x^m \cdot x^n = x^{m+n}$$

Example 2 Simplifying Expressions Using the Product Rule

Simplify.

(a.)
$$x^2yx^{-5}y^{-4}x^5x$$

SOLUTION

Simplify by adding the exponents of like bases.

$$x^{2}yx^{-5}y^{-4}x^{5}x$$

$$= x^{2}x^{-5}x^{5}xyy^{-4}$$

$$= x^{2+(-5)+5+1}y^{1+(-4)}$$

$$= x^{3}y^{-3}$$

 x^3y^{-3} may also be written with positive exponents as $\frac{x^3}{y^3}$.

b.
$$\frac{yy^{-3}x^4y^5x^{-10}}{y^{-6}x^{-3}y^{10}x^2}$$

SOLUTION

Step 1: Simplify the numerator and denominator by adding the exponents of like bases.

Step 2: Apply the definition of x^{-n} .

Step 3: Repeat step 1.

$$\frac{yy^{-3}x^4y^5x^{-10}}{y^{-6}x^{-3}y^{10}x^2} = \frac{x^{4+(-10)}y^{1+(-3)+5}}{x^{(-3)+2}y^{(-6)+10}} = \frac{x^{-6}y^3}{x^{-1}y^4} = x^{-6} \cdot x^1 \cdot y^3 \cdot y^{-4} = x^{-5}y^{-1} \text{ or } \frac{1}{x^5y^{-3}}$$

The product rule can be used to expand $(x^2)^3$ as

$$(x^2)^3 = x^2 x^2 x^2 = x^{2+2+2} = x^6$$

This demonstrates the power rule for exponents.

Power Rule for Exponents

If m, n, and x are real numbers,

$$(x^{m})^{n} = x^{m \cdot n}$$
.

This rule extends to any number of exponential factors, so that

$$(x^a y^b z^c \dots)^n = x^{an} y^{bn} z^{cn} \dots$$

Example 3 Simplifying Using the Power Rule of Exponents

Simplify.

a.
$$\frac{x(x^{-3})^2y(xy^{-2})^{-3}}{(x^2)^3y^{-3}(x^2)^3}$$

SOLUTION

Use the power rule in both the numerator and denominator, then simplify the expressions using the product rule.

$$\frac{x(x^{-3})^2y(xy^{-2})^{-3}}{(x^2)^3y^{-3}(x^2)^3} = \frac{xx^{-6}yx^{-3}y^6}{x^6y^{-3}x^6} = \frac{x^{-8}y^7}{x^{12}y^{-3}} = x^{-20}y^{10} = \frac{y^{10}}{x^{20}}.$$

b.
$$\frac{(x^a)^b(y^a)^{b+2}}{x^{-a}}$$

SOLUTION

$$\frac{(x^a)^b(y^a)^{b+2}}{x^{-a}} = \frac{x^{ab}y^{ab+2a}}{x^{-a}} = x^{ab}x^ay^{ab+2a} = x^{ab+a}y^{ab+2a}$$

Scientific notation is a method of writing a number as the product of a number greater than or equal to 1 but less than 10 and a power of 10. Multiplying a number by a positive integral power of 10 moves the decimal point to the right. Multiplying a number by a negative integral power of 10 moves the decimal point to the left.

Number	The decimal moves	Scientific Notation
412.36	Two places left	4.1236×10^{2}
0.041236	Two places right	4.1236×10^{-2}

Writing very large or small numbers in scientific notation can be helpful. Rules of exponents can be used to simplify expressions written in scientific notation.

Hint

The Commutative and Associative Properties of Multiplication allow you to move the numbers and to group them to make calculations easier.

Example 4 Simplifying Expressions in Scientific Notation

Simplify
$$\frac{(0.0003 \times 10^{-6})(4000)}{(0.006 \times 10^{15})(2000 \times 10^{4})}$$
.

SOLUTION

Begin by writing all four numbers in scientific notation. Then multiply and divide.

$$\frac{(3 \times 10^{-4} \times 10^{-6})(4 \times 10^{3})}{(6 \times 10^{-3} \times 10^{15})(2.0 \times 10^{3} \times 10^{4})} = \frac{(3 \times 10^{-10})(4 \times 10^{3})}{(6 \times 10^{12})(2 \times 10^{7})}$$
$$= \frac{3 \cdot 4}{6 \cdot 2} \times \frac{10^{-7}}{10^{19}} = 1 \times 10^{-7 + (-19)} = 1 \times 10^{-26}$$

Example 5 Astronomy

The speed of light is 3×10^8 meters/second. If the moon is 3.844×10^8 meters from the earth, how many seconds does it take light to reach the moon from the earth? Give the answer to the nearest hundredth.

SOLUTION Divide the moon's distance from the earth by the speed of light.

$$\frac{3.844 \times 10^8}{3 \times 10^8} = \frac{3.844}{3} \times \frac{10^8}{10^8} = \frac{3.844}{3} \approx 1.28 \text{ seconds}$$

It takes light about 1.28 seconds to reach the moon from the earth.

Lesson Practice

Simplify.

a.
$$\frac{1}{2^{-3}}$$

b.
$$2^{-4}$$

c.
$$-2^{-2}$$
 d. $(-2)^{-3}$

e.
$$-(-2)^{-4}$$
g. $\frac{xx^6y^8x^{-11}y^{-3}}{x^{-5}yy^2x^{-4}}$

i.
$$\frac{(x^b)^{a-1}(xy^{-a})^{-b}}{y^{-1}}$$

$$\mathbf{j.}_{(Ex\,4)} \frac{(0.004)(600 \times 10^9)}{(30000 \times 10^{-12})(0.0001 \times 10^3)}$$

k. The United States population is approximately 2.99×10^8 . In one year, the United States produced 4.78 × 1011 pounds of garbage. Approximately how much garbage did the average American produce that year?

Practice Distributed and Integrated

Evaluate each expression for the given values of a and b.

1.
$$-b^2 - b(a - b^2)$$
 if $a = 4$ and $b = -3$

1.
$$-b^2 - b(a - b^2)$$
 if $a = 4$ and $b = -3$ 2. $a^2 - b^3(a - b)$ if $a = -2$ and $b = -3$

State the properties illustrated below.

3.
$$a + b + c = b + a + c$$

4.
$$a(b+c) = ab + ac$$

Use the rules of exponents to simplify the expressions below.

*5.
$$\frac{(2x^2)^{-3}(xy^0)^{-2}}{2xx^0x^1xxy^2}$$

*6.
$$\frac{a^0bc^0(a^{-1}b^{-1})^2}{ab(ab^0)abc}$$

*7.
$$\frac{(2x^2y^3)^{-3}y}{(4xy)^{-2}(x^{-2}y)^3y}$$

*8.
$$\frac{xx^{-2}y(x^{-3})^2xy^0}{(2xy)^{-2}x^2(y^{-3})^2}$$

9. What is the area of a circle with radius 3 meters? Give your answer in terms of π .

- 10. Angles a and b are vertical angles. If angle b measures 74°, what is the measure of angle a?
- *11. Write In 2002, the average height of a 10-year old girl was 1.4×10^{-3} km. Approximately 3 × 107 10-year old girls lying head-to-toe would equal the circumference of the earth at the equator. Explain how you could use properties of real numbers to mentally estimate the circumference.
 - 12. Consumer Your dinner total at your favorite restaurant is \$18.
 - a. Use the Distributive Property to mentally calculate 10% of your dinner bill.
 - b. Explain how the result in part a would be helpful to mentally calculate a 15% tip.
 - 13. Error Analysis A student is asked to simplify an expression and identify which property was used for each step. His answer is shown below. Identify and explain the error(s).

$$(8 + 4) \cdot 9$$

 $(8 \cdot 9) + (4 \cdot 9)$ Associative Property of Addition

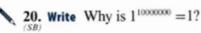
72 + 36Distributive Property

108 Add.

- 14. a. Evaluate the expression $1 (3x^2 2y)$ if x = 2 and y = 3.
 - b. Explain how the result would be affected if y was doubled.
 - **c.** Show that this is true by evaluating the expression if x = 2 and y = 6.
- 15. Nutrition The U.S. recommended daily amount of calcium for most adults is 1000 mg per day. Milana's breakfast consists of cereal that contains 33 mg calcium in $\frac{1}{4}$ -cup and milk that contains 75 mg calcium per $\frac{1}{4}$ -cup. Calculate the total amount of calcium in milligrams in Milana's breakfast of $\frac{3}{4}$ -cup cereal with $\frac{1}{2}$ -cup milk.
- 16. Justify A square drawn on a coordinate grid has adjacent vertices at points (u, 1) and (5u, 1). Another square has adjacent vertices at points (2u, 1) and (4u, 1). Write a simplified expression of their combined area. Explain how you obtained your answer.
- 2 17. Coordinate Geometry A figure on a coordinate graph is translated 2 units to the right and 5 units up. Its new coordinates can be written with the expressions (x + 2, y + 5). A point is translated to (19, 19), what are the coordinates of the original point?
 - 18. Verify Evaluate the expression $3g 2f + fg^2 5g + 4fg^2 + 6f$ if f = 1 and g = -1by inserting the variable values into the expression as written. Verify your answer by simplifying the expression first, then inserting the variable values.
 - *19. Multiple Choice Simplify $\frac{y^{-3}xy^2x^{-4}x^5}{x^3y^{-1}}$.

A $\frac{1}{x}$ **B** $\frac{1}{x^2 v^5}$ **C** $\frac{x^5}{v^2}$

 $\mathbf{D} = \frac{\chi^3}{V}$



*21. Error Analysis Two students incorrectly simplified the expression $\frac{2^3(2^{-2})^3}{2(2^2+2)}$. Their steps are shown below. Explain the error of each student.

Student A	Student B
$\frac{2^{3}(2^{-2})^{3}}{2(2^{2}+2)} = \frac{2^{3}(2^{-6})}{2(4+2)}$ $= \frac{2^{-3}}{12} = \frac{1}{12(-2)^{3}} = -\frac{1}{96}$	$\frac{2^{3}(2^{-2})^{3}}{2(2^{2}+2)} = \frac{2^{3}(2^{-6})}{2(2^{3})}$ $= \frac{2^{-3}}{2^{4}} = \frac{1}{2^{7}} = \frac{1}{128}$

- **22.** Volcanos Vailulu'u is an underwater volcano in the Pacific Ocean. The volcano has a height of about 4,200 m off the sea floor. If the top of the volcano is at a depth of 590 m below sea level, what is the depth of the sea floor in this area?
- *23. Verify The Atlantic Ocean is 3.93×10^3 m deep and has 8.24×10^{13} m² of surface area. Calculate the total volume of the Atlantic Ocean. Write your answer in scientific notation. Verify your answer if you know that the Pacific Ocean has approximately two times the volume and approximately the same depth of the Atlantic Ocean, and a surface area of about 1.66×10^{14} m².
- 24. Geography The Danube River is Europe's second longest river. It flows through 9 countries and has a length of 1,770 miles. About how many feet long is the Danube?
- *25. Geometry A box has a volume of $\frac{x^3y^2x^{-2}}{y^{-3}x^2y}$. The area resulting from length times width is $\frac{x^{-2}y^3x^{-1}}{y^{-4}x^{-2}y^2}$. Find an expression for the height of the box.
 - 26. Analyze Rita can choose to rent movies from one of two movie stores. The first store charges \$5 per movie, and she can rent her 8th movie for free. The second store charges \$7 per movie, and she can rent her 4th movie for free. Rita assumes that the second shop has a better deal because she must only rent 3 movies before receiving a free movie rental. Explain why she should not make this assumption.
- *27. Graphing Calculator Simplify $\frac{2^{-3}}{8^{-2}}$. Use a calculator to check your answer.
 - 28. Error Analysis The EPA lists the greenest hybrid car as getting 61 miles per gallon on the highway and 50 miles per gallon in the city. Assuming the fuel tank holds 15 gallons of gas, one owner decided to calculate the total miles that he could drive one tank of gas if 4 gallons were used in the city. Explain and correct his error.

$$61(4) + 50(15 - 4) = 244 + 75 - 200 = 119$$
 miles

29. Analyze Identify which property of real numbers is being demonstrated. Explain why this property might be helpful in solving this problem without a calculator.

$$(9 \cdot 3) \cdot 3 = 9 \cdot (3 \cdot 3) = 9 \cdot 9 = 81$$

- *30. Multi-Step Shane spent \$2.24 on breakfast and x dollars on lunch.
 - a. Write an expression for the amount of money Shane spent on breakfast and lunch.
 - b. If Shane spent \$5.97 on lunch, how much money did he spend in all?

SAXON° ALGEBRA 2

Structured for Results.

Saxon Secondary Mathematics is structured to help every student achieve success. The incremental development and continual practice and review provide the time needed to master each concept. Meaningful connections and real-world applications build a solid foundation and confidence to continue taking mathematics courses through calculus and beyond.

505822

Saxon Alg 2 4th Ed. Student Book MA \$128.53



