SAXON®
GEOMETRY

SAXON® GEOMETRY

Student Edition

SAXON®
HOUGHTON MIFFLIN HARCOURT
Supplemental Publishers

www.SaxonPublishers.com
800-531-5015

ISBN 13: 978-1-6027-7305-9
ISBN 10: 1-6027-7305-X

Printed in the United States of America

18 0868 17
4500699099

Table of Contents

Section 1: Lessons 1–10, Investigation 1

DISTRIBUTED STRANDS

- Geometry Foundations
- Logic and Reasoning
- Construction
- Coordinate Geometry
- Triangles: Congruence and Similarity
- Polygons
- Quadrilaterals
- Right Triangles and Trigonometry
- Circles
- Solids
- Transformations

Section 2: Lessons 11–20, Investigation 2

TABLE OF CONTENTS

Section 3: Lessons 21–30, Investigation 3

DISTRIBUTED STRANDS

- Geometry Foundations
- Logic and Reasoning
- Construction
- Coordinate Geometry
- Triangles: Congruence and Similarity
- Polygons
- Quadrilaterals
- Right Triangles and Trigonometry
- Circles
- Solids
- Transformations

Section 4: Lessons 31–40, Investigation 4

Section 5: Lessons 41–50, Investigation 5

DISTRIBUTED STRANDS

- Geometry Foundations
- Logic and Reasoning
- Construction
- Coordinate Geometry
- Triangles: Congruence and Similarity
- Polygons
- Quadrilaterals
- Right Triangles and Trigonometry
- Circles
- Solids
- Transformations

Section 6: Lessons 51–60, Investigation 6

Section 7: Lessons 61–70, Investigation 7

DISTRIBUTED STRANDS

- Geometry Foundations
- Logic and Reasoning
- Construction
- Coordinate Geometry
- Triangles: Congruence and Similarity
- Polygons
- Quadrilaterals
- Right Triangles and Trigonometry
- Circles
- Solids
- Transformations

Section 8: Lessons 71–80, Investigation 8

Section 9: Lessons 81–90, Investigation 9

DISTRIBUTED STRANDS

- Geometry Foundations
- Logic and Reasoning
- Construction
- Coordinate Geometry
- Triangles: Congruence and Similarity
- Polygons
- Quadrilaterals
- Right Triangles and Trigonometry
- Circles
- Solids
- Transformations

Section 10: Lessons 91–100, Investigation 10

Section 11: Lessons 101–110, Investigation 11

DISTRIBUTED STRANDS

- Geometry Foundations
- Logic and Reasoning
- Construction
- Coordinate Geometry
- Triangles: Congruence and Similarity
- Polygons
- Quadrilaterals
- Right Triangles and Trigonometry
- Circles
- Solids
- Transformations

Section 12: Lessons 111–120, Investigation 12

LESSON 1

Points, Lines, and Planes

Warm Up

Start off each lesson by practicing prerequisite skills and math vocabulary that will make you more successful with today's new concept.

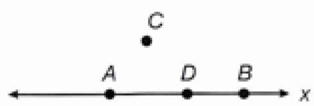
1. **Vocabulary** The __________ plane contains the x-axis and the y-axis. (SB 13)
2. Kira needs to buy a piece of pipe that is 40% longer than the 7-inch piece she already has. What length of pipe does Kira need? (SB 4)
3. Simplify $\sqrt[4]{81}$. (SB 6)
4. Evaluate $\frac{4(n + 6)}{2n}$ for $n = 2$. (SB 14)

New Concepts

In geometry, a **definition** of a term is a statement that defines a mathematical object. Definitions usually reference other mathematical terms. A basic mathematical term that is not defined using other mathematical terms is called an **undefined term**. In geometry, points, lines, and planes are undefined terms that are the building blocks used for defining other terms.

A **point** names a location and has no size. It is represented by a dot and labeled using a capital letter, such as P.

A **line** is a straight path that has no thickness and extends forever. There are an infinite number of points on a line. A line is named using either a lowercase letter or any two points on the line. Two possible names for the line shown in the diagram are $\overleftrightarrow{AB}$ and line x.

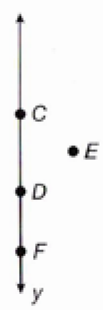


Hint

A ruler can be used to determine if points are collinear or noncollinear. A ruler can always connect two points, so two points are always collinear. Three points are only collinear if you can use the ruler to draw a line passing through all three of them.

Any set of points that lie on the same line are called **collinear** points. In the diagram, A, B, and D are collinear. If points do not lie on the same line, they are **noncollinear**. Points A, B, and C are noncollinear.

Example 1 Identifying Lines and Collinear Points



(a.) Give two different names for the line.

SOLUTION

Two possible names for the line are line y and $\overleftrightarrow{CD}$. The order of the points does not matter, so $\overleftrightarrow{DC}$ would also be correct.

(b.) Name three collinear points and three noncollinear points.

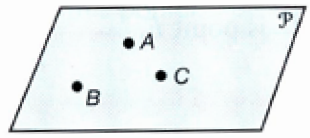
SOLUTION

Points C, D, and F are collinear. Points C, D, and E are noncollinear.

Online Connection www.SaxonMathResources.com

> **Math Reasoning**
>
> **Model** What are some common objects that are planes?

A **plane** is a flat surface that has no thickness and extends forever. A plane is named using either an uppercase letter or three noncollinear points that lie in the plane. The plane in the diagram below could be called plane $\mathcal{P}$ or plane *ABC*.



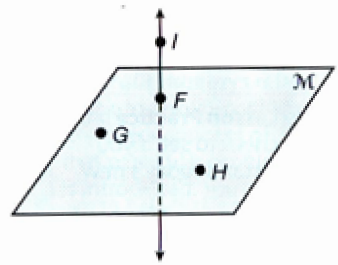
Lines or points that are in the same plane are said to be **coplanar**. If there is no plane that contains the lines or points, then they are **noncoplanar**. Space is the set of all points. Therefore, space includes all lines and all planes.

Example 2 Identifying Planes

What are two different names for this plane?

SOLUTION

Two possible names for the plane are plane *FGH* or plane $\mathcal{M}$.



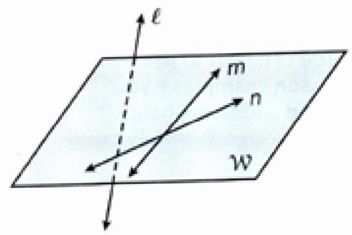
> Each day brings you a **New Concept** where a new topic is introduced and explained through thorough **Examples** — using a variety of methods and real-world applications.
>
> You will be reviewing and building on this concept throughout the year to gain a solid understanding and ensure mastery on the test.

Example 3 Identifying Coplanar Lines

(a.) Identify the coplanar and noncoplanar lines in the diagram.

SOLUTION

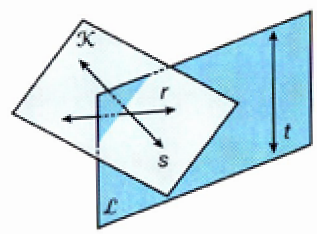
Lines *m* and *n* are coplanar. Line ℓ is noncoplanar with lines *m* and *n*.



(b.) Identify the coplanar and noncoplanar lines in the diagram.

SOLUTION

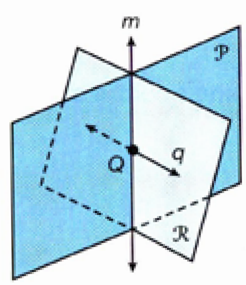
Lines *r* and *s* are coplanar. Line *t* is noncoplanar with lines *r* and *s*.



> **Math Reasoning**
>
> **Model** Can two planes have no intersections at all? What common objects illustrate what this might look like?

An **intersection** is the point or set of points in which two figures meet. When two lines intersect, their intersection is a single point. When two planes intersect, their intersection is a single line. If a line lies in a plane, then their intersection is the line itself. If the line does not lie in the plane, then their intersection is a single point.

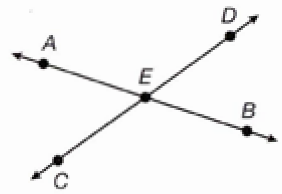
Lines *q* and *m* intersect at point *Q*. Plane $\mathcal{R}$ intersects plane $\mathcal{P}$ at line *m*. The intersection of plane $\mathcal{R}$ and line *m* is line *m*. Line *q* intersects planes $\mathcal{P}$ and $\mathcal{R}$ at point *Q*.



In some lessons, **Explorations** allow you to go into more depth with the mathematics by investigating math concepts with manipulatives, through patterns, and in a variety of other ways.

Example 4 Intersecting Lines and Planes

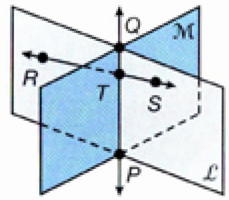
a. What is the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$?



SOLUTION

The intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ is point E.

b. What is the intersection of $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$?
What is the intersection of planes $\mathcal{M}$ and $\mathcal{L}$?



SOLUTION

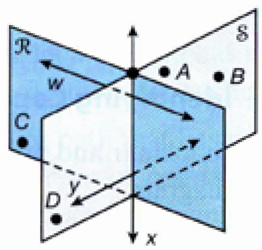
The intersection of $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ is point T. The intersection of the planes $\mathcal{M}$ and $\mathcal{L}$ is $\overleftrightarrow{PQ}$.

The **Lesson Practice** lets you check to see if you understand today's new concept.

The italic numbers refer to the Example in this lesson in which the major concept of that particular problem is introduced. You can refer to the lesson examples if you need additional help.

Lesson Practice

Identify each of the following from the diagram.



a. All of the lines. *(Ex 1)*

b. A pair of collinear points. *(Ex 1)*

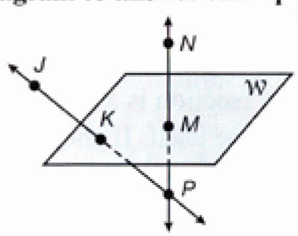
c. All of the planes. *(Ex 2)*

d. Three coplanar points. *(Ex 2)*

e. Two coplanar lines. *(Ex 3)*

f. A pair of noncoplanar lines. *(Ex 3)*

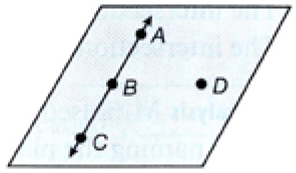
Use the diagram to answer each question.



g. What is the intersection of $\overleftrightarrow{JK}$ and $\overleftrightarrow{NM}$? *(Ex 4)*

h. What is the intersection of $\overleftrightarrow{JK}$ and plane $\mathcal{W}$? *(Ex 4)*
What is the intersection of $\overleftrightarrow{NP}$ and plane $\mathcal{W}$?

Practice Distributed and Integrated



1. In the diagram, which set of three points are collinear? Which point cannot be included in a collinear set of three points?
(1)

2. Can two points be noncollinear?
(1)

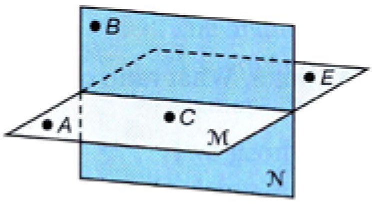
3. Can two noncoplanar lines intersect?
(1)

4. Name three undefined basic figures of geometry.
(1)

5. What term describes two lines that have a point in common?
(1)

6. **Generalize** Can three points be noncoplanar? Explain.
(1)

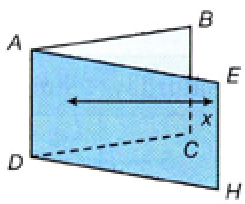
7. Name the coplanar points shown on plane $\mathcal{M}$.
(1)



The ***italic numbers*** refer to the lesson(s) in which the major concept of that particular problem is introduced. You can refer to the examples or practice in that lesson, if you need additional help.

8. **Write** The floor, ceiling, and walls of a room are all parts of planes. How many planes intersect the plane of the floor in your classroom? What geometric figures are formed where the planes intersect? Explain.
(1)

Use the diagram to answer problems 9–11.



9. What is the intersection between the planes?
(1)

10. **Justify** Are $\overleftrightarrow{AD}$, $\overleftrightarrow{CD}$, and $\overleftrightarrow{CH}$ coplanar? Explain.
(1)

11. State the intersection point of $\overleftrightarrow{BC}$ and line x.
(1)

12. **Multiple Choice** Which statement is true?
(1)
A Any two lines are coplanar.
B Any three lines are coplanar.
C Any two intersecting lines are coplanar.
D Any two perpendicular lines are noncoplanar.

13. How many different points can a line contain?
(1)

14. How many different lines can a plane contain?
(1)

15. **Multiple Choice** Which statement is true?
(1)
A The intersection of two lines forms a one-dimensional figure.
B The intersection of two planes forms a two-dimensional figure.
C The intersection of two planes forms a one-dimensional figure.
D The intersection of two lines forms a two-dimensional figure.

In the **Practice,** you will review today's new concept as well as math you learned in earlier lessons. By practicing problems from many lessons every day, you will begin to see how math concepts relate and connect to each other and to the real world.

Also, because you practice the same topic in a variety of ways over several lessons, you will have "time to learn" the concept and will have multiple opportunities to show that you understand.

16. **Error Analysis** Miri used two points to name a plane. What mistake did Miri make in naming the plane?
(1)

17. Evaluate: $5 - (7 + 8) \div 5 + (-2)^3$
(SB 1)

18. Name the property of addition shown by this equation:
(SB 2)
$(+3) + (-4) = (-4) + (+3)$

19. **Error Analysis** Jacob said that -3 is a rational number. Aaron said that -3 is an irrational number. Who is correct? Explain.
(SB 3)

20. **Justify** Is $0.\overline{3}$ irrational? Why or why not?
(SB 3)

21. **Baseball** A ball player is at bat 55 times and hits the ball 33 times. What ratio of his times at bat does he not hit the ball?
(SB 41)

22. Evaluate: $(-3)^3 - \left(\frac{1}{3}\right)^{-3}$
(SB 1)

23. Simplify: $2\sqrt{12} + 6\sqrt{27}$
(SB 6)

24. **Construction** A concrete pad has dimensions 9 feet by 9 feet by 4 inches. How many cubic yards of concrete does it contain?
(SB 9)

25. **Meteorology** Determine the mean and median values for the weekly rainfall data.
(SB 11)

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Rainfall (mm)	0.5	2	0	4	2.5	5	7

26. **Physics** On average, the acceleration due to gravity is 9.807 m/s^2. A science student measured it as 9.760 m/s^2. To the nearest hundredth of a percent, what was the student's percent error?
(SB 10)

27. **Coordinate Geometry** Plot these points on the coordinate plane: (2, 1), (−1, −1), (0, 0), and (3, −1).
(SB 13)

28. **Algebra** Evaluate the expression $xy^{-2} + \frac{x}{y}$, where $x = -2$, $y = \frac{1}{2}$.
(SB 14)

29. **Algebra** Transform the formula $I = Prt$ to solve for r.
(SB 16)

30. **Algebra** State the slope of the line $3x + 4y - 15 = 0$.
(SB 19)

The mixed set of Practice is just like the mixed format of your state test. You'll be practicing for the "big" test every day!

LESSON 2 Segments

Warm Up

1. (1) **Vocabulary** Points that lie on the same line are called ________ points.
2. (SB 15) Solve for x: $5x + 6 = 2x - 5$
3. (SB 1) Simplify: $5(2x - 6) + 3x - 7$

New Concepts

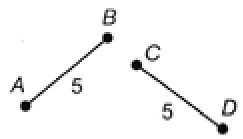
A **line segment** is a part of a line consisting of two **endpoints** and all points between them.

Math Language

Point *C* is **between** points *A* and *B* if *A*, *B*, and *C* are collinear and $AC + CB = AB$.

The diagram above depicts a line segment with endpoints A and B. A segment is named by its two endpoints in either order with a straight segment drawn over them. This segment could be called either $\overline{AB}$ or $\overline{BA}$.

Two geometric objects that have the same size and shape are **congruent**. **Congruent segments** have the same length.



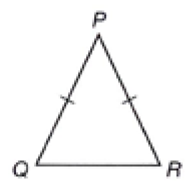
Caution

When comparing segments or other geometric figures, congruence statements are used. When the length of segments are being compared, or any other measurements that can be expressed as numbers, an equal sign is used.

In this figure, $\overline{AB}$ and $\overline{CD}$ are congruent. As shown on the diagram, they both have a **length** of 5 units.

A **congruence statement** shows that two segments are congruent. The symbol $\cong$ is read "is congruent to." The congruence statement for the segments above is $\overline{AB} \cong \overline{CD}$.

In a diagram, congruent segments are shown with **tick marks**. The diagram below shows congruent segments indicated by tick marks.



Hint

For more on the Reflexive, Symmetric, and Transitive Properties of Equality, see the Skills Bank at the back of this textbook.

The following properties apply to all congruent segments.

Reflexive Property of Congruence	$\overline{AB} \cong \overline{AB}$
Symmetric Property of Congruence	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
Transitive Property of Congruence	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Example 1 Using Properties of Equality and Congruence

Identify the property that justifies each statement.

a. $\overline{WX} \cong \overline{YZ}$, so $\overline{YZ} \cong \overline{WX}$

SOLUTION

Symmetric Property of Congruence

b. $\overline{PQ} \cong \overline{RS}$ and $\overline{RS} \cong \overline{TU}$, so $\overline{PQ} \cong \overline{TU}$

SOLUTION

Transitive Property of Congruence

c. $\overline{GH} \cong \overline{GH}$

SOLUTION

Reflexive Property of Congruence

Reading Math

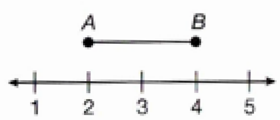
The length of $\overline{AB}$ is denoted AB.

A ruler can be used to measure the lengths of segments. The points on a ruler correspond with the points on a line segment. This concept is presented in the Ruler Postulate. A **postulate** is a statement that is accepted as true without proof.

Postulate 1: Ruler Postulate
The points on a line can be paired in a one-to-one correspondence with the real numbers such that: **1.** Any two given points can have coordinates 0 and 1. **2.** The distance between two points is the absolute value of the difference of their coordinates.

Distance is the measure of the segment connecting two points. The distance between two points can be represented by those two points with no segment symbol. For example, AB means "the distance between A and B."

Distance is always positive, so absolute values are used to calculate distances.



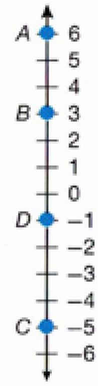
$|\text{Point } A - \text{Point } B| = |2 - 4| = |-2| = 2$

The distance from point A to point B is 2.

Online Connection
www.SaxonMathResources.com

Example 2 Finding Distance on a Number Line

Find each distance.



a. AB

SOLUTION

$AB = |6 - 3|$
$= |3|$
$= 3$

b. BC

SOLUTION

$BC = |3 - (-5)|$
$= |8|$
$= 8$

c. CD

SOLUTION

$CD = |(-5) - (-1)|$
$= |-4|$
$= 4$

d. AC

SOLUTION

$AC = |6 - (-5)|$
$= |11|$
$= 11$

Hint

Postulates are statements that are accepted as true without proof. See the Postulates and Theorems section in the back of this book for a complete list of postulates in this program.

In the example above, notice that $AC = AB + BC$. This is not a coincidence.

Postulate 2: Segment Addition Postulate
If B is between A and C, then $AB + BC = AC$.

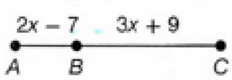
Example 3 Using the Segment Addition Postulate

a. Point S lies on $\overline{RT}$ between R and T. $RS = 12$ and $RT = 31$. Find ST.

SOLUTION

$RT = RS + ST$	Segment Addition Postulate
$31 = 12 + ST$	Substitute.
$19 = ST$	Subtract 12 from both sides.

b. Find AC in terms of x.

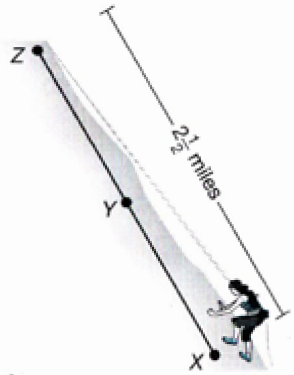


SOLUTION

$AC = AB + BC$	Segment Addition Postulate
$AC = (2x - 7) + (3x + 9)$	Substitute.
$AC = 5x + 2$	Simplify.

The **midpoint** of a segment is the point that divides the segment into two congruent parts. If M is the midpoint of $\overline{AB}$, then $AM = MB$.

Example 4 Application: Hiking



A hiker is traveling up a mountain towards the summit. The distance from the base of the mountain to the summit is 2.5 miles, as shown. How far will she have traveled when she reaches the midpoint (Y) of the hike?

SOLUTION

$XZ = XY + YZ$	Segment Addition Property
$XY = YZ$	Definition of midpoint
$XZ = XY + XY$	Substitute XY for YZ.
$2.5 = 2(XY)$	Substitute.
$1.25 = XY$	Divide both sides by 2.

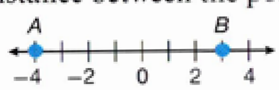
The hiker will have traveled 1.25 miles when she reaches the midpoint.

> **Hint**
>
> Notice in Examples 3 and 4 that each step of the solution is explained to the right. This will help prepare you for performing algebraic and geometric proofs later in the program.

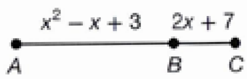
Lesson Practice

a. Identify the property that justifies the statement, $\overline{KL} \cong \overline{MN}$, so $\overline{MN} \cong \overline{KL}$. (Ex 1)

b. Find the distance between the points A and B. (Ex 2)



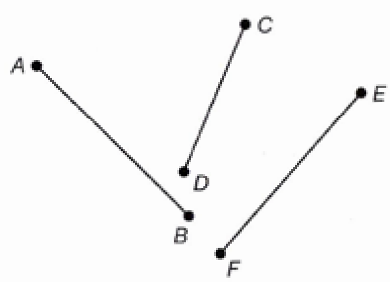
c. Find AC in terms of x. (Ex 3)



d. The drive from Seattle to San Francisco is 811 miles. How many miles is the midpoint from either city? (Ex 4)

Practice Distributed and Integrated

1. Measure the line segments at right to determine which segments are congruent. (2)



2. Estimate $\sqrt{32}$ to the nearest tenth. (SB 6)

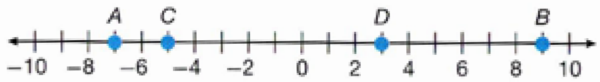
3. What property is illustrated by this statement? (SB 2)

If a and b are real numbers, then $a + b = b + a$.

4. (1) **Multiple Choice** What is the least number of points that can determine a plane?

A 1 **B** 2
C 3 **D** 4

Calculate the length of each segment using the diagram below.



5. (2) $\overline{AD}$

6. (2) $\overline{BC}$

7. (2) $\overline{DA}$

8. (2) $\overline{AC}$

9. (SB 11) **Data Analysis** The numbers in this set are the math test scores for a class. Find the mode.

$$\{62, 53, 74, 55, 66, 72, 80, 83, 83, 86, 92, 93, 40, 51, 61, 71\}$$

10. (SB 14) Evaluate $3m^2 - 5m + 17$ for $m = -3$.

11. (SB 13) In what quadrant is point S if its coordinates are $(2, -4)$?

12. (SB 11) Find the median of the set $\{2, 2, 3, 5, 6, 7, 7, 8, 9\}$.

13. (2) **Algebra** A, B, and C are collinear, $AB = 5x - 19$, and $BC = 3x + 4$. Find an expression for AC if B is between A and C.

14. (2) **City Planning** The city is planning to install streetlights and wants five lights along a walkway of 60 yards. If there is a light at the beginning and at the end of the walkway and the lights are evenly spaced, what is the distance between each light?

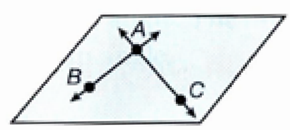
15. (1) **Error Analysis** Sunil stated that three points determine a unique plane. Explain Sunil's error and and give a corrected statement.

16. (2) **Algebra** Suppose $AB = 3x$, $BC = 2y + 16$, $AC = 60$, and B is the midpoint of AC. Find the values of x and y.

17. (2) **Analyze** Points D, E, and F are collinear with E between D and F. $DE = 15$, $EF = x + 17$, and $DF = 3x - 10$. Find EF and DF.

18. (1, 2) **Write** Describe how equality and congruence are used to describe two line segments and their lengths.

19. (1) Are points A, B, and C collinear, coplanar, both, or neither?



20. (1) How many points are required to determine a line?

21. (1) **Multiple Choice** Which of these statements is false?

A Two distinct points determine a line.
B Three noncollinear points determine a plane.
C Two noncoplanar lines determine a space.
D Four noncoplanar points determine a space.

22. Lines MN and PQ intersect at point E. Name two sets of three collinear points.
(1)

23. Factor: $2x^2 - 16x - 66$
(SB 18)

24. Expand: $(x - 4)(x + 7)$
(SB 18)

25. **Carpentry** A piece of wood that is 8 feet long needs to be 7 feet 4 inches long. How much has to be cut off? Express your answer in feet.
(SB 9)

26. Simplify: $\frac{-36x^{-4}y^5}{12x^2y^{-3}}$. Express your answer with positive exponents.
(SB 6)

27. **Sales Tax** If sales tax is 6%, estimate the sales tax on an item that retails for \$48.99.
(SB 4)

28. Estimate the sum to the nearest whole number.
(SB 5)
$1.8 + 2.345 + 0.65 + 13.56$

29. **Multi-Step** There are 1000 liters in a cubic meter and approximately 3.85 liters in one gallon. A swimming pool measures 8 meters wide by 4 meters long by 1.5 meters deep. Approximately how many gallons of water can the pool hold?
(SB 9)

30. **Physics** In a free fall, acceleration due to gravity is approximately 9.8 m/s^2. Convert this rate into ft/s^2.
(SB 9)

LESSON 3

Angles

Warm Up

1. (2) **Vocabulary** Two figures that have the same size and shape are called __________ figures.

2. (SB 1) Simplify: $\frac{1}{2}\left(\frac{2^2}{2} - 2\right)$

3. (SB 3) Convert $\frac{22}{7}$ into a decimal number. Is it a terminating decimal number, repeating decimal number, or non-terminating and non-repeating decimal number?

New Concepts

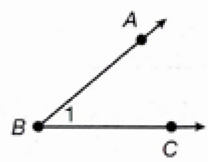
A **ray** is a part of a line that starts at an endpoint and extends infinitely in one direction.

A ray is named by its endpoint and any other point on the ray. For example, the ray in the diagram is called $\overrightarrow{AB}$, which is read "ray AB."

Two rays that have a common endpoint and form a line are called **opposite rays**.

Rays $\overrightarrow{YX}$ and $\overrightarrow{YZ}$ are opposite rays.

An **angle** is a figure formed by two rays with a common endpoint. The common endpoint is the angle's **vertex**. The rays are the **sides** of the angle. The sides of this angle are $\overrightarrow{BA}$ and $\overrightarrow{BC}$. The vertex is B.

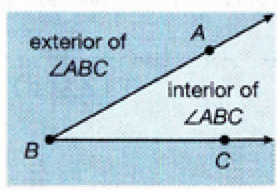


Caution

An angle can be named by its vertex only if it is clear that there is only one angle at the vertex.

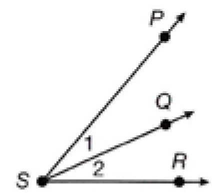
An angle can be named in several different ways: by its vertex, by a point on each ray and the vertex, or by a number. For example, the angle in the diagram could be called $\angle B$, $\angle ABC$, $\angle CBA$, or $\angle 1$.

The exterior of an angle is the set of all points outside the angle. The interior of an angle is the set of all points between the sides of an angle.



Online Connection
www.SaxonMathResources.com

Example 1 Naming Angles and Rays



a. Name three rays in the diagram.

SOLUTION

$\overrightarrow{SP}$, $\overrightarrow{SQ}$, and $\overrightarrow{SR}$

b. Name three angles in the diagram.

SOLUTION

$\angle PSQ$ or $\angle 1$, $\angle QSR$ or $\angle 2$, and $\angle PSR$.

c. Could $\angle PSQ$ also be referred to as $\angle S$?

SOLUTION

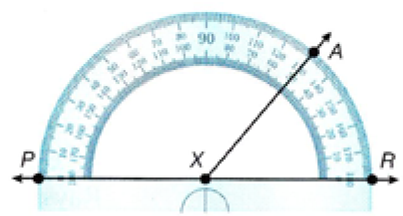
No, there are three different angles with S as a vertex.

A **protractor** is a tool used to measure angles. Unlike segments, angles are measured in **degrees**. One degree is a unit of angle measure that is equal to $\frac{1}{360}$ of a circle.

Postulate 3: Protractor Postulate

Given a point X on $\overleftrightarrow{PR}$, consider rays $\overrightarrow{XP}$ and $\overrightarrow{XR}$, as well as all the other rays that can be drawn with X as an endpoint, on one side of $\overleftrightarrow{PR}$. These rays can be paired with the real numbers from 0 to 180 such that:

1. $\overrightarrow{XP}$ is paired with 0, and $\overrightarrow{XR}$ is paired with 180.
2. If $\overrightarrow{XA}$ is paired with a number c and $\overrightarrow{XB}$ is paired with a number d then $m\angle AXB = |c - d|$.



Math Language

Analyze Rays that form a **straight angle** are called opposite rays.

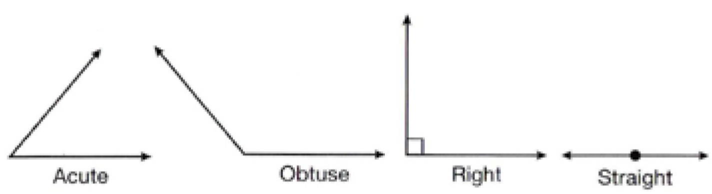
Angles are classified according to their angle measure.

An **acute angle** measures greater than 0° and less than 90°.

An **obtuse angle** measures greater than 90° and less than 180°.

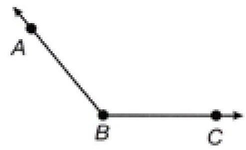
A **right angle** measures exactly 90°. A box drawn at the vertex of an angle shows that it is a right angle, as shown in the diagram.

A **straight angle** measures exactly 180°.



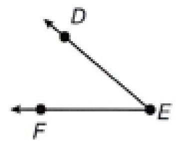
Example 2 Measuring and Classifying Angles

a. Use a protractor to measure $\angle ABC$, then classify the angle.



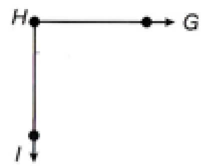
SOLUTION $\angle ABC$ measures 130°, so it is an obtuse angle.

b. Use a protractor to measure $\angle DEF$, then classify the angle.



SOLUTION $\angle DEF$ measures 40°, so it is an acute angle.

c. Use a protractor to measure $\angle GHI$, then classify the angle.



SOLUTION $\angle GHI$ measures 90°, so it is a right angle.

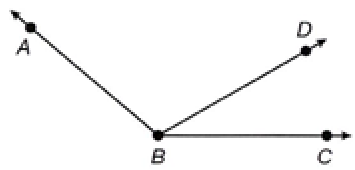
Angles can be added in the same way that segments are added.

> **Reading Math**
>
> The measure of $\angle ABD$ is represented by adding a lowercase 'm' to the notation, that is $m\angle ABD$.

Postulate 4: The Angle Addition Postulate

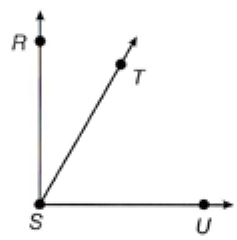
If point D is in the interior of $\angle ABC$, then

$m\angle ABD + m\angle DBC = m\angle ABC.$



Example 3 Using the Angle Addition Postulate

The measure of $\angle RST = 22°$ and $m\angle TSU = 69°$. Find $m\angle RSU$. Classify the angle.



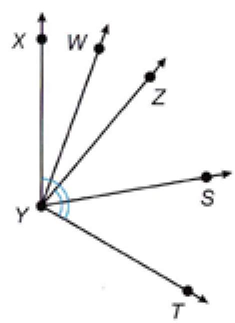
SOLUTION

$m\angle RST + m\angle TSU = m\angle RSU$	Angle Addition Postulate
$22° + 69° = m\angle RSU$	Substitute.
$91° = m\angle RSU$	Simplify.

$\angle RSU$ is an obtuse angle.

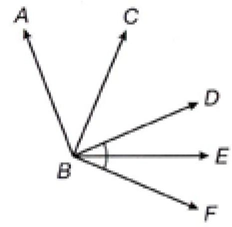
To **bisect** a figure is to divide it into two congruent parts. An **angle bisector** is a ray that divides an angle into two **congruent angles**. Congruent angles have the same measure. They are marked with **arc marks**, as shown in the diagram.

$\angle XYW \cong \angle WYZ$ and $\angle ZYS \cong \angle SYT$.



Example 4 Using Angle Bisectors and Congruence Marks

The measure of $\angle ABC = 44°$. $\overrightarrow{BC}$ bisects $\angle ABD$. The measure of $\angle EBF = 23°$. Find the measure of $\angle CBE$.



SOLUTION Since $\overrightarrow{BC}$ bisects $\angle ABD$, it divides $\angle ABD$ into two congruent angles. So, $\angle ABC \cong \angle CBD$ and $m\angle ABC = m\angle CBD$. Since $m\angle ABC = 44°$, $m\angle CBD = 44°$.

Using the congruence marks in the diagram, $\angle DBE \cong \angle EBF$, so $m\angle DBE = m\angle EBF$. Since $m\angle EBF = 23°$, $m\angle DBE = 23°$.

$m\angle CBE = m\angle CBD + m\angle DBE$	Angle Addition Postulate
$= 44° + 23°$	Substitute.
$= 67°$	Add.

The measure of $m\angle CBE$ is 67°.

Example 5 Application: Interpreting Statistics

Hint

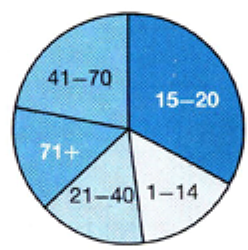
For more about displaying data using circle graphs, see the Skills Bank at the back of this textbook.

Louis runs a restaurant. He knows that he has about 900 customers a day. The circle graph in the diagram shows what percentage of his customers fall into the given age brackets. He wants to know exactly how many of his customers are between ages 15 and 20. Use a protractor to measure the angle and find the number of Louis's customers that fall into the 15–20 age bracket.

SOLUTION

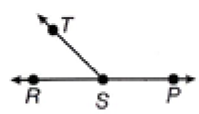
Measure the angle of the sector that represents 15–20-year-old customers. The sector has an angle measure of 120°. Since an entire circle is 360°, this is $\frac{120}{360} = \frac{1}{3}$ of the circle. One third of Louis's customers is $\left(\frac{1}{3}\right)(900) = 300$ customers.

Customers by Age

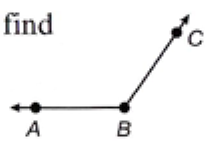


Lesson Practice

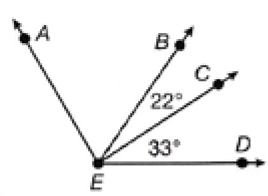
a. Name three rays and three angles in the diagram.
(Ex 1)



b. Classify $\angle ABC$ and use a protractor to find the measure of it.
(Ex 2)



c. Determine $m\angle AEB$ if $m\angle AED = 120°$.
(Ex 3)



SAXON®
GEOMETRY

Structured for Results.

Saxon Secondary Mathematics is structured to help every student achieve success. The incremental development and continual practice and review provide the time needed to master each concept. Meaningful connections and real-world applications build a solid foundation and confidence to continue taking mathematics courses through calculus and beyond.

ISBN-13: 978-1-602-77305-9
90000>
9 781602 773059
1270742

SAXON®
HOUGHTON MIFFLIN HARCOURT