INTERMEDIATE LOGIC

Mastering Propositional Arguments

STUDENT: THIRD EDITION

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INTERMEDIATE LOGIC

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INTRODUCTION

Logic has been defined both as the *science* and the *art* of correct reasoning. People who study different sciences observe a variety of things: biologists observe living organisms, astronomers observe the heavens, and so on. From their observations they seek to discover natural laws by which God governs His creation. The person who studies logic as a science observes the mind as it reasons—as it draws conclusions from premises—and from those observations discovers laws of reasoning which God has placed in the minds of people. Specifically, he seeks to discover the principles or laws which may be used to distinguish good reasoning from poor reasoning. In deductive logic, good reasoning is *valid* reasoning—in which the conclusions follow necessarily from the premises. Logic as a science discovers the principles of valid and invalid reasoning.

Logic as an *art* provides the student of this art with practical skills to construct arguments correctly as he writes, discusses, debates, and communicates. As an art logic also provides him with rules to judge what is spoken or written, in order to determine the validity of what he hears and reads. Logic as a science discovers rules. Logic as an art teaches us to apply those rules.

Logic may also be considered as a symbolic language which represents the reasoning inherent in other languages. It does so by breaking the language of arguments down into symbolic form, simplifying them such that the arrangement of the language, and thus the reasoning within it, becomes apparent. The outside, extraneous parts of arguments are removed like a biology student in the dissection lab removes the skin, muscles and organs of a frog, revealing the skeleton of bare reasoning inside. Thus revealed, the logical structure of an argument can be examined, judged and, if need be, corrected, using the rules of logic.

So logic is a symbolic language into which arguments in other languages may be translated. Now arguments are made up of propositions, which in turn are made up of terms. In categorical logic, symbols (usually capital letters) are used to represent terms. Thus "All men are sinners" is translated "All M are S." In propositional logic, the branch of logic with which this book primarily deals, letters are used to represent entire propositions. Other symbols are used to represent the logical operators which modify or relate those propositions. So the argument, "If I don't eat, then I will be hungry; I am not hungry, so I must have eaten" may appear as $\sim E \supset H, \sim H, \therefore E$.

Unit 1 of this book covers the translation and analysis of such propositional arguments, with the primary concern of determining the validity of those arguments. Unit 2 introduces a new kind of logical exercise: the writing of formal proofs of validity and related topics. Unit 3 completes propositional logic with a new technique for analyzing arguments: truth trees. Unit 4 considers how to apply these tools and techniques to arguments contained in real-life writings: philosophy, theology, and the Bible itself. Unit 5 introduces digital logic and helps students to unlock the logic of electronic devices.

UNIT 1

TRUTH TABLES

CONTENTS

LESSON 1

INTRODUCTION TO PROPOSITIONAL LOGIC

Propositional logic is a branch of formal, deductive logic in which the basic unit of thought is the propositon. A **proposition** is a statement, a sentence which has a truth value. A single proposition can be expressed by many different sentences. The following sentences all represent the same proposition:

God loves the world.
The world is loved by God.
Deus mundum amat.

These sentences represent the same proposition because they all have the same meaning.

In propositional logic, letters are used as symbols to represent propositions. Other symbols are used to represent words which modify or combine propositions. Because so many symbols are used, propositional logic has also been called "symbolic logic." Symbolic logic deals with **truth-functional propositions**. A proposition is truth-functional when the truth value of the proposition depends upon the truth value of its component parts. If it has only one component part, it is a **simple proposition**. A categorical statement is a simple proposition. The proposition *God loves the world* is simple. If a proposition has more than one component part (or is modified in some other way), it is a **compound proposition**. Words which combine or modify simple propositions in order to form compound propositions (words such as *and* and *or*) are called **logical operators**.

For example, the proposition *God loves the world and God sent His Son* is a truth-functional, compound proposition. The word *and* is the logical operator. It is truth functional because its truth value depends upon the truth value of the two simple propositions which make it up. It is in fact a true proposition, since it is true that God

DEFINITIONS

Propositional logic is a branch of formal, deductive logic in which the basic unit of thought is the proposition. A ***proposition*** is a statement.

KEY POINT

One proposition may be expressed by many different sentences.

DEFINITIONS

A proposition is ***truth-functional*** when its truth value depends upon the truth values of its component parts.

If a proposition has only one component part, it is a ***simple proposition***. Otherwise, it is ***compound***.

DEFINITIONS

Logical operators are words that combine or modify simple propositions to make compound propositions.

A ***propositional constant*** is an uppercase letter that represents a single, given proposition.

A ***propositional variable*** is a lowercase letter that represents any proposition.

loves the world, and it is true that God sent His Son. Similarly, the proposition *It is false that God loves the world* is compound, the phrase *it is false that* being the logical operator. This proposition is also truth-functional, depending upon the truth value of the component *God loves the world* for its total truth value. If *God loves the world* is false, then the proposition *It is false that God loves the world* is true, and vice versa.

However, the proposition *Joe believes that God loves the world,* though compound (being modified by the phrase *Joe believes that),* is *not* truth-functional, because its truth value does not depend upon the truth value of the component part *God loves the world.* The proposition *Joe believes that God loves the world* is a self-report and can thus be considered true, regardless of whether or not *God loves the world* is true.

When a given proposition is analyzed as part of a compound proposition or argument, it is usually abbreviated by a capital letter, called a **propositional constant**. Propositional constants commonly have some connection with the propositions they symbolize, such as being the first letter of the first word, or some other distinctive word within the proposition. For example, the proposition *The mouse ran up the clock* could be abbreviated by the propositional constant M. On the other hand, *The mouse did not run up the clock* may be abbreviated *~M* (read as *not M*). Within one compound proposition or argument, the same propositional constant should be used to represent a given proposition. Note that a simple proposition cannot be represented by more than one constant.

When the *form* of a compound proposition or argument is being emphasized, we use **propositional variables**. It is customary to use lowercase letters as propositional variables, starting with the letter *p* and continuing through the alphabet (*q*, *r*, *s*, . . .). Whereas a propositional constant represents a single, given proposition, a propositional variable represents an unlimited number of propositions.

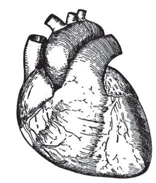
It is important to realize that a single constant or variable can represent not only a simple proposition but also a compound proposition. The variable *p* could represent *God loves the world* or it could represent *God loves the world but He hates sin.* The entire compound

KEY POINT

A propositional constant or variable can represent a simple proposition or a compound proposition.

proposition *It is false that if the mouse ran up the clock, then, if the clock did not strike one, then the mouse would not run down* could be abbreviated by a single constant F, or it could be represented by symbolizing each part, such as $\sim(M \supset (\sim S \supset \sim D))$. The decision concerning how to abbreviate a compound proposition depends on the purpose for abbreviating it. We will learn how to abbreviate compound propositions in the next few lessons.

SUMMARY



A proposition is a statement. Propositions are truth-functional when the truth value of the proposition depends upon the truth value of its component parts. Propositions are either simple or compound. They are compound if they are modified or combined with other propositions by means of logical operators. Propositional constants are capital letters which represent a single given proposition. Propositional variables are lower case letters which represent an unlimited number of propositions.

EXERCISE 1 (25 points)

What are two main differences between propositional constants and propositional variables?

1. ______________________________

2. ______________________________

Modify or add to the simple proposition *We have seen God* to create the following:

3. A truth-functional compound proposition:

   ____

4. A proposition which is *not* truth-functional:

   ____

Circle S if the given proposition is simple. Circle C if it is compound.

5. The Lord will cause your enemies to be defeated before your eyes. S C
6. There is a way that seems right to a man but in the end it leads to death. S C
7. The fear of the Lord is the beginning of wisdom. S C
8. If we confess our sins then He is faithful to forgive us our sins. S C
9. It is false that a good tree bears bad fruit and that a bad tree bears good fruit. S C
10. The Kingdom of God is not a matter of talk but of power. S C

Given that		
	B means *The boys are bad*	M means *The man is mad*
	G means *The girls are glad*	S means *The students are sad*

Translate the following compound propositions:

11. It is false that B. ______________________________
12. B or G. ______________________________
13. B and M.______________________________
14. If M then S. ______________________________
15. If not M and not S then G.______________________________

    ____

LESSON 2

NEGATION, CONJUNCTION, AND DISJUNCTION

We will begin our study of abbreviating and analyzing compound propositions by learning about three fundamental logical operators: *negation, conjunction,* and *disjunction.* As we do, we will be answering three questions for each logical operator: What words in English are abbreviated by it? What is its symbol? How is the truth value of the compound proposition affected by the truth values of the component parts?

KEY POINT

Three fundamental logical operators are **negation**, **conjunction**, and **disjunction**.

Negation

Negation is the logical operator representing the words *not, it is false that,* or any other phrase which denies or contradicts the proposition. As we have already seen, the symbol ~ (called a *tilde*) represents negation. If the proposition *All roads lead to Rome* is represented by the propositional constant *R*, then ~*R* means *Not all roads lead to Rome* or *It is false that all roads lead to Rome.* Note that the negation of a proposition is the contradiction of that proposition. Thus ~*R* could also be translated *Some roads do not lead to Rome*. If a proposition is true, its negation is false. If a proposition is false, its negation is true. This can be expressed by the following **truth table**, where T means *true* and F means *false*:

p	~p
T	F
F	T

Truth tables show how the truth value of a compound proposition is affected by the truth value of its component parts. The table above is called the **defining truth table** for negation because it completely defines its operations on a minimum number of variables (in this case, one). The defining truth table for an operator that joins two propositions would require two variables.

DEFINITIONS

Negation (~, *not*) is the logical operator that denies or contradicts a proposition.

A ***truth table*** is a listing of the possible truth values for a set of one or more propositions. A ***defining truth table*** displays the truth values produced by a logical operator modifying a minimum number of variables.

DEFINITIONS

Conjunction (•, *and*) is a logical operator that joins two propositions and is true if and only if both the propositions (*conjuncts*) are true.

Conjunction

When two propositions are joined by *and, but, still,* or other similar words, a **conjunction** is formed. The conjunction logical operator is symbolized by • (called, of course, a *dot*). If *Main Street leads to home* is represented by the constant *H*, then *All roads lead to Rome, but Main Street leads to home* could be represented by *R* • *H* (read as *R dot H,* or *R and H*).

The conjunction is true if and only if its components (called **conjuncts**) are both true. If either conjunct is false, the conjunction as a whole is false. The defining truth table for conjunction is therefore:

p	q	p • q
T	T	T
T	F	F
F	T	F
F	F	F

Thus if *All roads lead to Rome* is false and *Main Street leads to home* is true, then the entire conjunction *All roads lead to Rome but Main Street leads to home* is false, as seen on the third row down.

In ordinary English, the conjunction is not always placed between two distinct sentences. For example, *Paul and Apollos were apostles* could be symbolized *P* • *A*, where *P* means *Paul was an apostle* and A means *Apollos was an apostle.* Similarly, the proposition *Jesus is both God and man* could be represented by *G* • *M*.

DEFINITIONS

Disjunction (∨, *or*) is a logical operator that joins two propositions and is true if and only if one or both of the propositions (*disjuncts*) is true.

Disjunction

A **disjunction** is formed when two propositions are joined by the logical operator *or*, as in *Paul was an apostle or Apollos was an apostle.* The symbol for disjunction is ∨ (called a *vee*). The foregoing disjunction would thus be symbolized *P* ∨ *A* (read simply *P or A*).

In English, the word *or* is ambiguous. In one sense it can mean "this or that, but not both" (called the *exclusive or*). For example, in the sentence *The senator is a believer or an unbeliever,* the word *or* must be taken in the exclusive sense; nobody could be both a believer and an unbeliever at the same time in the same way. However, the word *or* can also mean "this or that, or both" (called the *inclusive or*). This is how it should be taken in the sentence *Discounts are given either* *to senior citizens or war veterans.* If you were a senior citizen or a war veteran *or both,* you would be allowed a discount.

In Latin, the ambiguity is taken care of by two separate words: *aut,* meaning the "exclusive *or,*" and *vel,* meaning the "inclusive *or.*" Although it may seem like the exclusive sense of the word *or* is the more natural sense, in logic the disjunction is always taken in the inclusive sense. This is seen in the fact that the symbol $\vee$ is derived from the Latin *vel.*

The defining truth table for disjunction is therefore:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

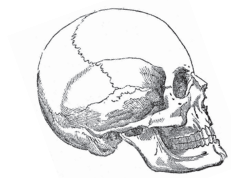
A disjunction is thus considered to be false if and only if both components (called **disjuncts**) are false. If either disjunct is true, the disjunction as a whole is true.

If the context of an argument requires that the word *or* be represented in the exclusive sense, as in *The senator is either a Republican or a Democrat,* it may be translated with the more complicated $(R \vee D) \bullet \sim(R \bullet D)$—that is, "The senator is either a Republican or a Democrat, but not both a Republican and a Democrat." However, you should assume that *or* is meant in the more simple inclusive sense unless instructed otherwise.

As you can see, logic may use parentheses in symbolizing complicated compound propositions. This is done to avoid ambiguity. The compound proposition $A \vee B \bullet C$ could mean *A or B, and C* or it could mean *A, or B and C.* Parentheses remove the ambiguity, as in $(A \vee B) \bullet C$, which represents *A or B, and C.* This is similar to how parentheses are used in mathematics. Assuming there are no rules about which operation should be performed first, the mathematical expression $5 + 6 \times 4$ could equal either 44 or 29, depending on whether one adds first or multiplies first. But parentheses would make it clear, as in $(5 + 6) \times 4$. Logic uses parentheses in the same way. Generally, in a series of three or more connected propositions, parentheses should be used.

KEY POINT

The logical operator for disjunction is always understood in the inclusive sense: "this or that, or both." If you intend the exclusive *or,* you must specify it explicitly.



CAUTION

Though in English grammar the word *or* is called a conjunction, in logic only *and* (and equivalent words) is a conjunction. *Or* is always called a disjunction.

KEY POINT

Generally, in a series of three or more connected propositions, parentheses should be used to avoid ambiguity.

CAUTION

Do not confuse the propositional meaning of the phrases *not both* and *both not.* Use parentheses to distinguish between them.

KEY POINT

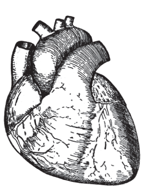
In the absence of parentheses, assume that negation attaches only to the proposition it immediately precedes.

The word *both* is often an indicator of how parentheses are to be placed when using conjunctions. The symbolized *exclusive or* in the paragraph above could be read *R or D, but not **both** R and D,* the word *both* telling us to place parentheses around $R \bullet D$.

A proper use of parentheses can also help us to distinguish between *not both* and *both not* propositions. For example, the proposition *Cats and snakes are not both mammals* (which is true) would be symbolized as $\sim(C \bullet S)$. The *not* comes before the *both,* so the tilde is placed before the parenthesis. However, *Both cats and snakes are not mammals* (which is false) would be symbolized as $(\sim C \bullet \sim S)$. Note that this second proposition could also be translated *Neither cats nor snakes are mammals.*

When symbolizing compound propositions which use negation, it is standard practice to assume that whatever variable, constant, or proposition in parentheses the tilde immediately precedes is the one negated. For example, the compound proposition $\sim p \vee q$ is understood to mean $(\sim p) \vee q$, because the tilde immediately precedes the variable p. This is different from $\sim(p \vee q)$. Negation is used in the same way that the negative sign is used in mathematics. The mathematical expression $-5 + 6$ means $(-5) + 6$, which equals 1. This is different from $-(5 + 6)$, which equals -11. So when negating a single variable or constant, you need not use parentheses. But when negating an entire compound proposition, place the tilde in front of the parentheses around the proposition.

SUMMARY



Three common logical operators are negation (*not*, symbolized $\sim$), conjunction (*and*, symbolized $\bullet$), and disjunction (*or*, symbolized $\vee$). These logical operators can be defined by means of truth tables. Negation reverses the truth value of a proposition, conjunction is true if and only if both conjuncts are true, and disjunction is false if and only if both disjuncts are false.

EXERCISE 2 (26 points)

Given: J means *Joseph went to Egypt* F means *There was a famine*
I means *Israel went to Egypt* S means *The sons of Israel became slaves*

Translate the symbolic propositions.

1. $F \bullet I$ ______________________________
2. $\sim J \vee S$ ______________________________
3. $\sim(J \vee I)$ ______________________________
4. $J \bullet \sim S$ ______________________________

Symbolize the compound propositions.

5. Joseph and Israel went to Egypt. ____________
6. Israel did not go to Egypt. ____________
7. Israel went to Egypt, but his sons became slaves. ____________
8. Either Joseph went to Egypt, or there was a famine. ____________
9. Joseph and Israel did not both go to Egypt. ____________
10. Neither Joseph nor Israel went to Egypt. ____________
11. Joseph and Israel went to Egypt; however, there was a famine, and the sons of Israel became slaves. ____________
12. Israel went to Egypt; but either Joseph did not go to Egypt, or there was a famine. ____________

LESSON 3

TRUTH TABLES FOR DETERMINING TRUTH VALUES

So far we have seen that truth tables help define logical operators. Truth tables also serve other functions, one of which is to help us determine the truth value of compound propositions. The truth value of elementary negations, conjunctions and disjunctions can be immediately determined from their defining truth tables. But what about compound propositions like $\sim p \lor (\sim q \bullet r)$? To find the truth values for such complicated propositions, the following procedure may be followed:

KEY POINT

Truth tables help determine the truth value of compound propositions, given the truth value of their component parts.

1. Draw a line, and on the leftmost part of the line place the variables (or constants) which are used in the proposition. Under these, put all the possible combinations of true and false. This will require four rows for two variables, eight rows for three variables, and in general 2^n rows for n variables. Under the first variable, place a T for each of the first half of the rows, then an F for each of the second half. Under the next variable, place half again as many Ts, half again as many Fs, then repeat this. The final column should have alternating single Ts and Fs, as follows:

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

You can verify for yourself that all the possible combinations of true and false are found in these eight rows.

KEY POINT

When completing a truth table, start with the standard truth values for the variables (or constants), then find the truth values for the negated variables (or constants).

2. If any variables are negated, these should be added next, with the corresponding truth values under them (specifically, under the operator):

↓ ↓

p	q	r	~p	~q
T	T	T	F	F
T	T	F	F	F
T	F	T	F	T
T	F	F	F	T
F	T	T	T	F
F	T	F	T	F
F	F	T	T	T
F	F	F	T	T

Here arrows are placed over *p* and *q* to show that those basic variables are being used to build more complicated propositions on the right-hand side of the table. Whenever *p* is true, *~p* is false, and vice versa, just as the defining truth table for negation shows. This is also the case for *q* and *~q*.

KEY POINT

After determining truth values for negations, complete the truth values for compound propositions within parentheses.

3. Continue to the next level of complexity in the proposition. As in mathematics, whatever is in parentheses should be completed before going outside the parentheses. In our example, the proposition in parentheses is *~q* • *r*. This is placed on the line, and whenever both *~q* and *r* are true, the conjunction *~q* • *r* is true, according to the defining truth table for conjunction. Thus we now have:

↓ ↓

p	q	r	~p	~q	(~q • r)
T	T	T	F	F	F
T	T	F	F	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	F

4. Continue with the same procedure, adding on to the truth table until the entire compound proposition is filled out. In our example, the propositions *~p* and *(~q • r)* are disjuncts. Thus, whenever