Second Edition

Tests

Algebra and Trigonometry

Functions and Applications

Paul A. Foerster

Tests for Algebra and Trigonometry

Second Edition

Functions and Applications

by

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Addison-Wesley Publishing Company
Menlo Park, California · Reading, Massachusetts · Don Mills, Ontario
Wokingham, England · Amsterdam · Sydney · Singapore · Tokyo
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Contents

Description of Tests

Addison-Wesley's *Algebra and Trigonometry* tests are adaptations of tests developed for use in the author's own classes. The tests are appropriate for use with both the school edition (Code No. 25086) and the college edition (Code No. 23253) of Paul A. Foerster's *Algebra and Trigonometry* text. Included is a test for each chapter, plus more detailed tests to cover material up to logical breaking points within chapters. Also included are 4 two-hour cumulative tests and a three-hour final exam that correspond to the cumulative reviews and final exam in the text.

Because most instructors prefer to write their own tests, the tests provided here are intended to guide you in writing your own tests. However, they are reproducible and may be used if the need arises and if they fit your requirements.

These tests can also be used as make-up tests for students who miss the regularly scheduled ones. If a photocopier is not available, you can give the test page to the student with instructions not to write on it. For this reason answers are separate from the tests.

Another way to create make-up (or regular!) tests is to copy problems that are examples in the text. This tactic speeds review after the test. You can say, for instance, "That was the example on page 117." If you use examples from the text a time or two, students will soon learn to read the book and work the examples before a test. Also, students cannot claim "We never saw that before!" if it was a part of their assigned reading.

It is not expected that you will give your students all 49 tests. To allow for selection of topics, there is more material in the text than can be presented in a one-year course. Obviously, you will test only the topics you have presented. Also, you will often include material from the last few sections of a chapter on the chapter test rather than giving a separate test over this material. In keeping with this idea of continuing review, most of these tests also contain material from earlier in the course. It is recommended that your own test program include this feature so that students do not forget things they once knew how to do.

It is intended that students show their work on notebook paper rather than on the test sheet. Also, it is expected that that you will evaluate students' work, not simply check their answers. However, consideration has been given to ease of grading of the tests. For example, problems that require students to draw graphs are placed at the end of the test whenever possible, so that you will not have to shuffle back and forth between sheets of notebook paper and graph paper.

It is expected that students will use calculators on all tests. No consideration of using computers has been given in writing these tests. See the Supplementary Classroom Exercises for ideas on evaluation of students' work done on the computer.

You are invited to give comments to the author concerning ideas for making these tests better. Write care of Addison-Wesley Publishing Co., 2725 Sand Hill Road, Menlo Park, CA 94025.

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Test 1 | CHAPTER 1

In first-year algebra you learned about variables, expressions, equations, and inequalities. The domain of a variable is the set of numbers to which it belongs. Give an example of

1. a negative rational number that it not an integer.

2. a positive irrational number that is not transcendental.

3. an imaginary number.

4. Explain why 35.21 is a *rational* number.

5. Evaluate the expression $3x^2 - 6x + 7$ if
a. x is 5. b. x is -4.

6. Solve the equation $3x - 7 = -40$ if the domain of x is
a. {integers} b. {rational numbers} c. {counting numbers}

7. Solve and graph the inequality $|2 - 3x| > 17$.

One kind of expression is the *polynominal.*

8. Name by degree and number of terms: $7^3x^4y^5 + z^8$

9. Name by degree and number of terms: $5x^4 - 7x^3 + 11$

10. Write an example of a quadratic monomial.

11. Write an example of an expression that is *not* a polynomial.

One technique that helps evaluate expressions is simplifying them. Simplify the following:

12. $20 - 12 \div 3 \cdot 2 + 1$

13. $|14 - 31| - 8$

14. $7 - 5[4 - 3(2 + x)]$

15. $5[2x - \frac{1}{4}(20x - 3)]$

16. Multiply: $(3x - 2)(5x + 4)$

17. Factor: $x^2 - 11x - 12$

The Field Axioms provide reasons for each step in a simplification. Use variables to state

18. the cummutative axiom for multiplication.

19. the associative axiom for addition.

20. the additive inverse axiom.

21. closure under multiplication.

Name the Field Axiom that was used:

22. $x + (y + z) = x + (z + y)$

23. $x(y + z) = xy + xz$

24. State and prove the multiplication property of equality.

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Test 2 | CHAPTER 2

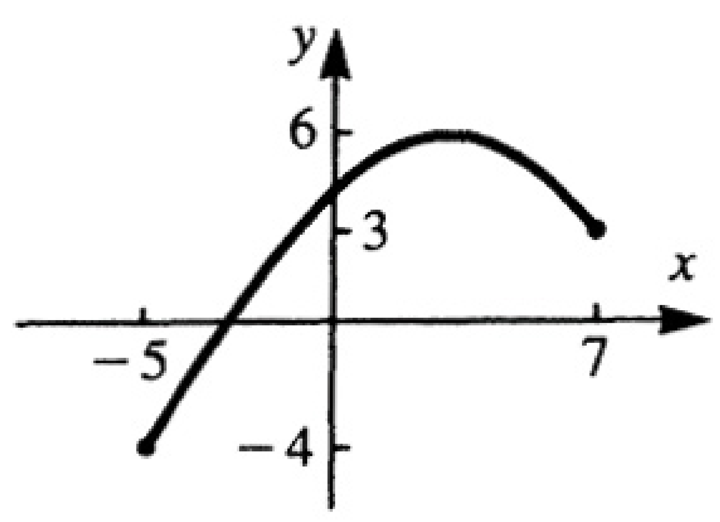
Algebra II is a study of functions. A function tells how two variables are related to each other. An equation may specify how they are related. For Problems 1, 2, and 3, plot the graph of the function. Tell its range.

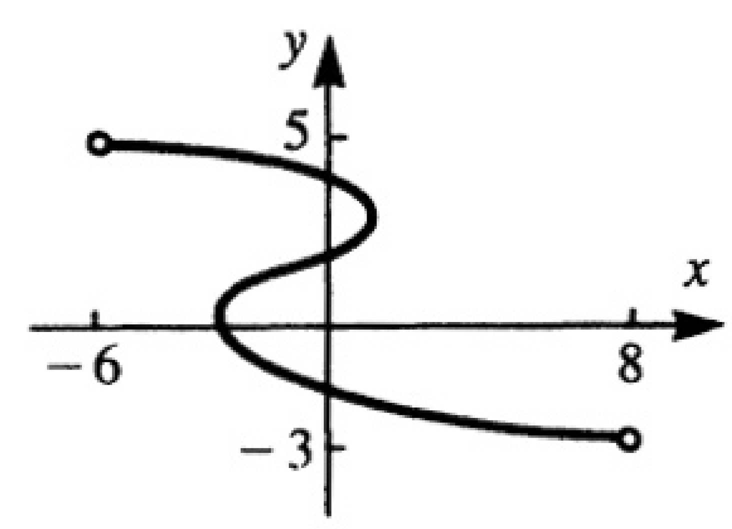
1. $y = 0.2x^3$, domain $= \{x: -3 \le x \le 3\}$

2. $y = x - 1$, domain $= \{x: -2 < x < 4\}$

3. $y = |x - 3|$, domain $= \{\text{digits}\}$

A relation may or may not be a function. For Problems 4 and 5, tell the domain, the range, and whether or not it is a function.

4. 

5. 

6. Plot the graph of the relation $|y| = x + 2$. Tell what the domain of x must be in order for there to be values of y. Tell whether or not the relation is a function.

Functions may relate two variables in the real world. For Problems 7, 8, and 9, sketch a reasonable graph.

7. The depth of water in a lake behind a dam depends on the number of days since they started letting water out through the flood gates.

8. The grade you could make on an algebra test is a function of the length of time you study for it.

9. The number of miles an airplane has flown on a given trip and its altitude above sea level are related.

For Problems 10 and 11, sketch the graph of the function described.

10. The function has domain $\{x: 3 < x < 8\}$ and range $\{y: -2 < y < 5\}$.

11. The graph has a vertical asymptote at $x = 4$.

It is important to be able to work old problems.

12. Plot a number-line graph: $|x + 5| \ge 2$

13. Name by degree and number of terms: $7^3x^4y^5 + z^8$

14. Evaluate: $\frac{3}{4} + \frac{1}{3}$

15. Simplify: $7 - 5[4 - 3(2 + x)]$

16. Solve: $(x + 3)(2x - 5) = 0$

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Test 3 | CHAPTER 3, Sections 3-1 to 3-4

Algebra/Trig is a study of functions. The first kind of function you are studying is the linear function.

1. Write the general equation for a linear function.

For Problems 2 and 3, transform to slope-intercept form.

2. $3x - 7y = 42$

3. $y - 5 = -\frac{1}{2}(x - 3)$

Functions have *intercepts*.

4. Write the definition of x-intercept.

5. If $3x - 5y = 90$, find the y-intercept.

6. If $4x + 7y = 56$, find the x-intercept.

You should be able to plot the graph of a linear function *quickly* by using properties you have learned. For Problems 7 through 10, plot the graph on graph paper.

7. $y = -\frac{2}{5}x + 3$

8. $y + 6 = 2(x - 4)$

9. $y = 3$

10. $x - 4y = 12$

By using properties in the reverse direction, you should be able to find the particular equation of a linear function if you know information about the graph. For Problems 11 through 14, find the particular equation of the linear function described. Leave your answer in the most convenient form.

11. Line contains (2, 9) and (5, 4).

12. Line is parallel to the graph of $3x + 4y = -17$, y-intercept $= 5$.

13. Line contains the origin, and is perpendicular to the graph of $y = 0.25x + 11$.

14. Line is vertical, and contains (5, 8).

It is important to remember old techniques.

15. Sketch the graph of a relation that is *not* a function.

16. Write a negative even integer less than -10.

17. You drink a glass of root beer through a straw. The amount of root beer left depends on the number of sips you have taken. Sketch a reasonable graph.

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Test 4 | CHAPTER 3, Section 3-5

Algebra/Trig is a study of functions (and relations). One of the simplest kinds of function is the linear function. On notebook paper, sketch a reasonable graph of the following linear functions:

1. Positive slope, negative y-intercept.

2. Domain: 3 to 7, inclusive; Range: 1 to 4, inclusive.

3. The amount of bismuth salicylate in a bottle of Pepto-Bismol varies linearly with the amount of Pepto-Bismol left in the bottle.

Linear functions can be used as mathematical models of relationships between two variables in the real world.

4. *Reading Problem* Phoebe Small has 35 pages of history to be read after she has been reading for 10 minutes, and 5 pages left after she has been reading for 50 minutes. Assume that the number of pages left varies linearly with the number of minutes she has been reading. Write the particular equation expressing pages in terms of minutes.

5. *Donuts Problem* The price you pay for a box of donuts varies linearly with the number of donuts in the box. For 5 donuts the price is \$1.15, and for 11 donuts it is \$2.35.
   a. Write the particular equation expressing price in terms of number of donuts.
   b. Predict the price of a box containing 3 donuts.
   c. If a box costs \$3.15, how many donuts would you expect it to contain?
   d. Sketch the graph of this linear function.
   e. Tell the real-world meanings of the slope and the price-intercept.

6. *Elevator Problem* The number of feet of cable needed for an elevator depends on the number of stories in the building it serves. Suppose that $c = 20s + 35$, where c is the number of feet of elevator cable and s is the number of stories.
   a. How do you know that c varies *linearly* with s?
   b. How much cable is needed for a 29-story building?
   c. How tall a building needs 375 feet of cable?
   d. What does the slope represent in the real world?
   e. What does the c-intercept equal? Why do you suppose that it is greater than zero?
   f. Write a suitable domain for the linear function.
   g. On graph paper, plot the graph of this function, observing the domain you wrote in part (f).

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Test 5 | CHAPTER 3

Algebra/Trig is a study of functions (and relations). One of the simplest kinds of function is the linear function.

1. Write the general equation that defines a linear function.

The particular equation of a linear function can be found from information about its graphs. Find the particular equation.

2. Contains (3, 5) and (−2, 7).

3. Has slope $-\frac{3}{7}$ and contains (4, −1).

4. Has x-intercept 6 and y-intercept 9.

5. Is vertical and contains (4, 6).

6. Is horizontal and contains (3, 7).

The equation of a linear function can be transformed to other forms.

7. Transform $4x - 8y = 13$ to slope-intercept form.

8. What is the slope of the line perpendicular to the one in Problem 7 if it goes through the point (−11, 17)?

9. $y - 5 = \frac{7}{3}(x + 4)$ is in "point-slope" form.
   a. Write the coordinates of the "point."
   b. Transform to $Ax + By = C$ form, where A, B, and C are integer constants.
   c. Find the x-intercept.

While you are learning new things, it is important to remember old techniques.

10. Tell which axiom was used: If x and y are real numbers, then xy is a unique real number.

11. Solve the equation: $|12x - 71| = 11$

12. Multiply: $(3x + 4)(2x - 7)$

Linear functions can be used as mathematical models of relationships between two variables in the real world.

13. Suppose that a baby sitter charges \$3.50 an hour, plus \$2.00 fixed charge. Write the particular equation for this function. Then use it to predict the number of hours sat for a charge of \$21.25.

14. Linear functions can be graphed quickly. Plot the graph of the linear function in Problem 9.

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Test 6 | CHAPTER 4, Sections 4-1 to 4-3

Algebra/Trig is a study of functions. The intersection point of two linear function graphs can be found by solving a system of equations.

1. Solve by linear combination (addition-subtraction).

$$\begin{aligned} 3x + 7y &= -29 \\ 4x - 9y &= 53 \end{aligned}$$

2. Solve by determinants (Cramer's rule).

$$\begin{aligned} 5x + 2y &= 8 \\ 3x - 4y &= -6 \end{aligned}$$

3. On notebook paper, sketch the graphs of two linear equations that are clearly *independent*.

4. Try to solve by linear combination. Then sketch a graph showing what went wrong.

$$\begin{aligned} 3x - 5y &= 15 \\ 3x - 5y &= 20 \end{aligned}$$

5. Try to solve by determinants. Then sketch a graph showing what went wrong.

$$\begin{aligned} 8x + 12y &= 36 \\ 6x + 9y &= 27 \end{aligned}$$

While you are learning new things it is important to remember old techniques.

6. Write a negative decimal that is also a rational number. Then write the decimal in a form that shows *why* it is a rational number.

7. Suppose that $y^2 = x + 5$. Evaluate y when x is 11. Then explain why this relation is *not* a function.

8. On notebook paper, sketch the graph of a function with domain from 6 to 9 (*not* inclusive), and range from 1 to 7 (*not* inclusive).

9. *Computer Disk Problem* A computer store sells 10 floppy diskettes for \$15, and 30 diskettes for \$40. Assume that the number of dollars is a linear function of the number of diskettes. Write the particular equation expressing dollars in terms of diskettes.

10. On graph paper, plot the graph of $y = -\frac{2}{5}x - 3$.

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Test 7 | CHAPTER 4, Sections 4-4 to 4-9

Algebra/Trig is a study of functions. In dealing with functions you must sometimes solve systems of equations with three variables.

1. Solve by augmented matrices. Express the answers as decimals.

$$\begin{aligned} 2x + 7y &= 8 \\ 5x - 3y &= 11 \end{aligned}$$

2. Solve by linear combination.

$$\begin{aligned} 3x + 2y - z &= 3 \\ x + 4y + 2z &= 19 \\ 2x + 3y - 5z &= 0 \end{aligned}$$

3. Solve the system in Problem 2 by augmented matrices.

4. Use third-order determinants (Cramer's rule) to find the value of y in Problem 2, thus showing that you get the same answer.

In mathematics it is important to remember how to solve old problems.

5. Draw a number-line graph of $|4x - 25| > 13$ if the domain of x is {natural numbers}.

When you deal with more than one function, it is convenient to use "$f(x)$" terminology.

6. Let $g(x) = 3x - 4$ and $h(x) = 2x^3$. Find $g(3)$, $h(5)$, $h(g(1))$, and $g(g(8))$.

$f(x)$ terminology is useful in some real-world problems.

7. *Swimming Problem* Adolph Finn is 130 meters from the beach. He swims toward the beach at 3.2 meters per second (m/sec). Let x be the number of seconds since he started swimming, and let $f(x)$ be the number of meters he still has to go to reach the beach.
   a. Write an equation expressing $f(x)$ in terms of x.
   b. Find $f(0)$ and $f(20)$.
   c. Sheila Rive is only 80 meters from the beach. At the same time Adolph starts, Sheila starts swimming toward the beach at 1.7 m/sec.
      Let $g(x)$ be the number of meters Sheila still has to go to reach the beach.
      Write an equation expressing $g(x)$ in terms of x.
   d. Find $g(0)$ and $g(20)$.
   e. Calculate the time at which $f(x) = g(x)$.
   f. Does Sheila arrive at the beach before Adolph reaches her? Explain.
   g. Plot the graphs of functions f and g on the same Cartesian coordinate system.

8. Three-variable linear equations have graphs that are planes in space. Draw the graph of $3x + 2y + 5z = 30$.

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Test 8 | CHAPTER 4, Sections 4-10 and 4-11

Algebra/Trig is a study of functions. $f(x)$ terminology can be used to define functions.

1. Given $f(x) = 3x^2 - 5x + 7$, find $f(6)$ and $f(-4)$.

Systems of linear inequalities can be used to define relations.

2. Plot on graph paper the solution set of the system

$$3x - 2y \geq 12 \text{ and } y < \frac{1}{5}x + 2.$$

A system of inequalities can be used to define the domain of two independent variables in a function with three variables.

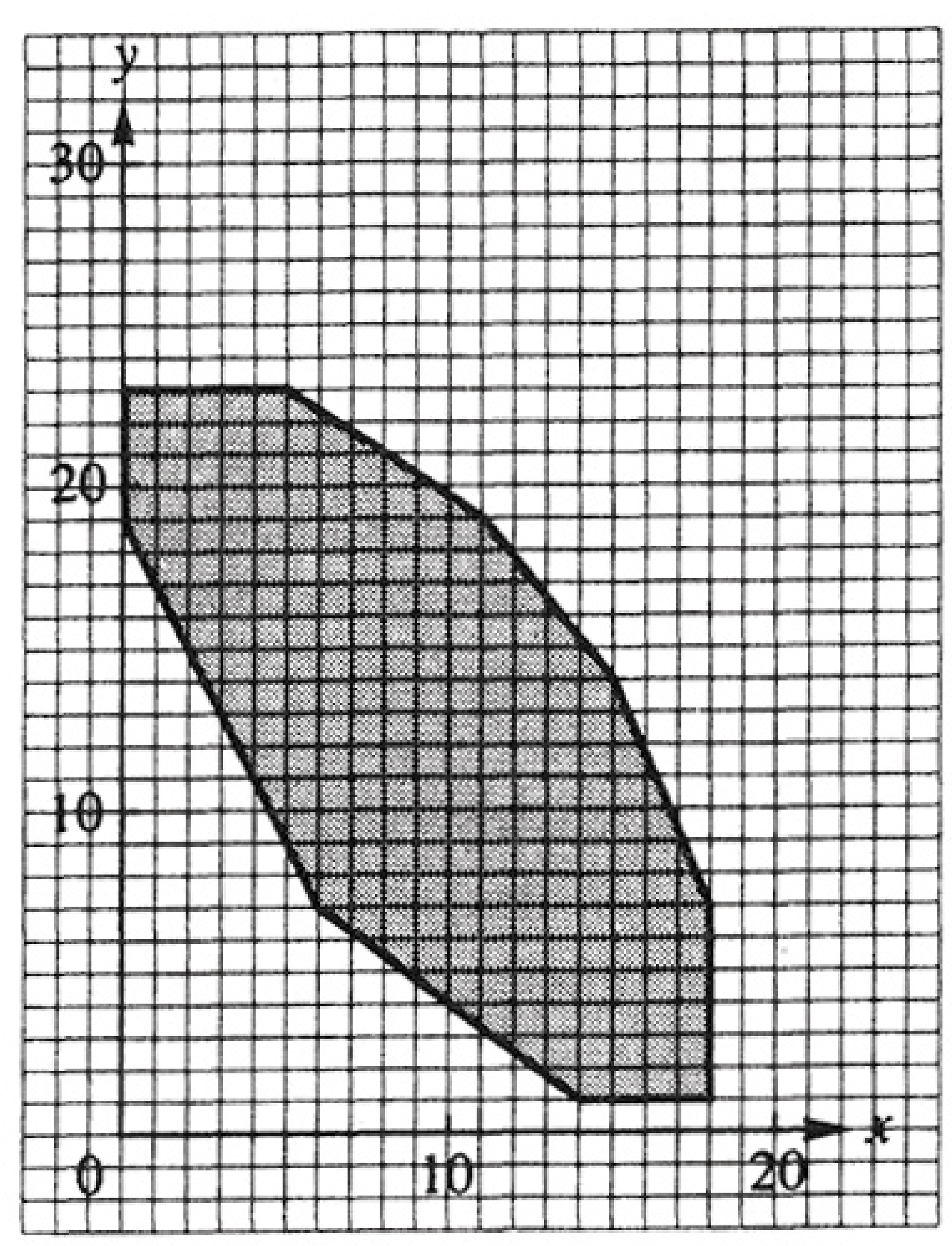
3. *Clothing Sale Problem* The Student Council decides to raise money by selling imported clothes. Let x be the number of shirts imported, and let y be the number of dresses imported. The import order must meet the following requirements.

i. The total number of shirts and dresses must be at least 10.
ii. The number of shirts must be between 5 and 27, inclusive.
iii. The number of dresses must be less than twice the number of shirts, but at least 1/6 the number of shirts.
iv. Four times the number of shirts plus three time the number of dresses must be at most 135.

Write the inequalities described, and plot the graph of the feasible region.

4. The diagram shows the feasible region for a manufacturing process in which x is the number of large machines made and y is the number of small machines made. The profit for each large machine is $1000, and the profit for each small machine is $500.

a. Write an equation for the profit in terms of x and y.
b. Shade the part of the feasible region in which the profit is at least $15,000.
c. What is the optimum number of small and large machines that produces the maximum profit? What is the maximum profit?



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Test 9 | CHAPTER 4

Algebra/Trig is a study of functions. Dealing with functions sometimes involves systems of equations or with two or more variables.

1. Solve by determinants.

$$\begin{aligned} 3x - 7y &= 13 \\ 2x + 9y &= -4 \end{aligned}$$

2. Solve by linear combination.

$$\begin{aligned} 5x - 4y &= -47 \\ 7x + 6y &= 27 \end{aligned}$$

3. Solve by augmented matrices.

$$\begin{aligned} 5x + 2y &= 8 \\ 3x + y &= 6 \end{aligned}$$

4. Solve.

$$\begin{aligned} 3x - 4y &= 9 \\ 6x - 8y &= 1 \end{aligned}$$

5. Solve by linear combination.

$$\begin{aligned} 2x + 3y - z &= -1 \\ -x + 5y + 3z &= -10 \\ 3x - y - 6z &= 5 \end{aligned}$$

6. Write the system in Problem 5 as an augmented matrix. Transform the matrix so that there are 0's in the lower left triangle. Use the result to find the value of z.

7. Find the value of z in Problem 5 by determinants.

8. Given $f(x) = 3x - 7$ and $g(x) = 4x + 5$, find the ordered pair where the graphs of f and g cross.

Three-variable linear equations have graphs that are planes in space.

9. Draw the graph of $3x + 4y + 2z = 12$.

Linear inequalities have graphs that are regions in the Cartesian plane.

10. Given the following requirements:
 i. The sum of x and y is at most 10.
 ii. y is less than twice x.
 iii. y is at least 2 more than $\frac{1}{3}$ of x.

Write inequalities expressing each of these requirements. Then plot the graph of the solution of this system.

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Test 10 | CHAPTER 5, Sections 5-1 to 5-5

Algebra/Trig is a study of functions. One kind of function you study is the quadratic function.

1. If $f(x) = 5x^2 - 8x + 13$,
  a. evaluate $f(3)$.
  b. evaluate $f(-7)$.

To evaluate x when $f(x)$ is given, you must solve quadratic equations with one variable.

2. Solve by the quadratic formula: $5x^2 - 8x - 3 = 0$

Quadratic functions have graphs that are parabolas. Sketch on notebook paper the graph of the quadratic function described.

3. Vertex at $(5, -7)$ and y-intercept 20.

4. y-intercept -3, and *no* real x-intercepts.

5. Exactly *one* x-intercept, $x = 4$.

If the equation of a quadratic function is in vertex form, you can easily sketch the graph.

6. Sketch on notebook paper the graph of $y - 8 = -3(x + 4)^2$.

If the equation is not in vertex form, you can transform it by completing the square.

7. Transform to vertex form: $y = 7x^2 + 42x - 13$

The x-intercepts of a quadratic function can be found by using the definition of intercept in connection with some of the above techniques.

8. Find the x-intercepts of $y = x^2 - 13x - 3$.

Some quadratic equations have solutions that are not real numbers.

9. Does $3x^2 + 7x + 5 = 0$ have real-number solutions? Justify your answer.

10. Given the quadratic equation $x^2 - 12x + 52 = 0$,
  a. Solve it over the set of complex numbers.
  b. Plot the two solutions in the complex plane.
  c. Write the word that describes how the two solutions are related to each other.
  d. Check one of the two solutions by substituting it into the original equation.

While you are studying quadratic functions, it is important not to forget about other techniques.

11. Find the particular equation expressing y in terms of x for the linear function containing $(-1, -2)$ and $(3, 10)$.

12. Do the squaring: $(5x + 11)^2$

13. Factor: $x^2 - 3x - 10$

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Test 11 | CHAPTER 5, Sections 5-6 and 5-7

Algebra/Trig is a study of functions. The particular equation of a quadratic function can be found from points on its graph.

1. Find the particular equation of the quadratic function containing $(-2, -11)$, $(4, 13)$, and $(6, 29)$.

2. Find the particular equation of the parabola with vertex at $(2, -5)$ and passing through $(3, 1)$.

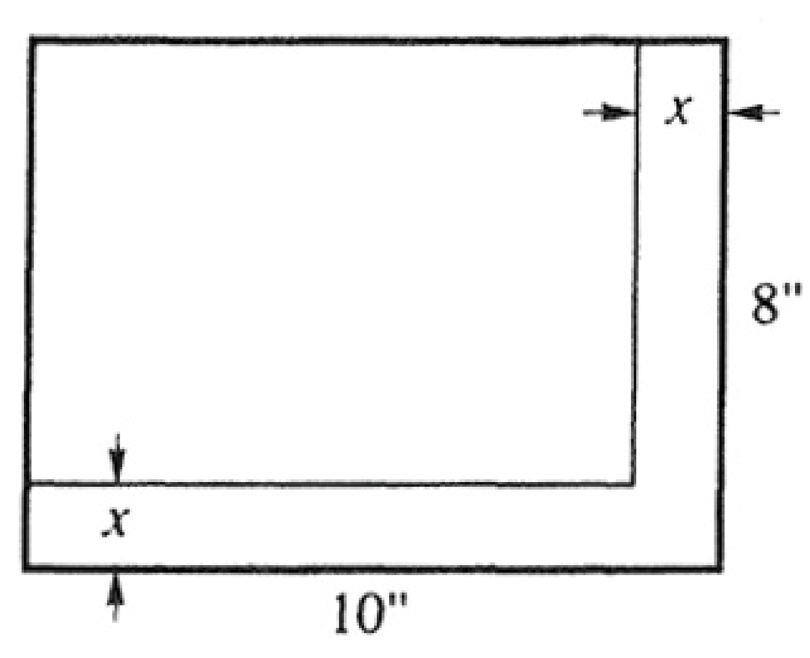
Quadratic functions can be used as mathematical models.

3. *Pizza Problem* Pete Zarilla sells 10-inch pizzas for \$4.30, 12-inch pizzas for \$5.88, and 14-inch pizzas for \$7.78. Assume that the price is a quadratic function of the diameter.

a. Write the particular equation expressing price in terms of diameter.
b. Predict the price of a 5-inch mini-pizza.
c. If a pizza sells for \$20.00, what would you expect its diameter to be?
d. What is the significance of the fact that the price-intercept is greater than zero?

Equations of quadratic functions can be found from geometrical information.

4. An L-shaped strip x inches wide is cut from a piece of cardboard (see sketch). Write the particular equation for the area of the remaining piece of cardboard as a function of x.



It is important to remember how to work old problems.

5. Solve the inequality $17 - 4x > 28$ and graph on a number line.

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Test 12 | CHAPTER 5

Algebra/Trig is a study of functions. In this chapter you have learned about quadratic functions.

1. Given: $y = 3x^2 + 24x + 43$
   a. Transform to vertex form.
   b. Write the coordinates of the vertex.
   c. Find the x-intercepts.
   d. Sketch the graph, showing vertex, x- and y-intercepts, and symmetrical point.

Solving quadratic equations sometimes leads to imaginary and complex numbers.

2. Solve $7x^2 - 5x - 3 = 0$.

3. Solve $x^2 + 2x + 26 = 0$.

4. Multiply $(5 + 7i)(4 - 3i)$.

5. Given $f(x) = 11x^2 + 13x - 28$, use the discriminant to determine whether or not $f(x)$ ever equals -100 for any real values of x.

While working with quadratic functions, you must not forget about linear functions.

6. Sketch the graph of a linear function whose domain is 3 to 10, inclusive, and whose range is 1 to 4, inclusive.

Quadratic functions can be used as mathematical models.

7. *Vertical Motion Problem* Chuck Stone is standing atop a high platform. He fires a rock up into the air with his slingshot. While it is in flight, the rock's distance above the ground is a quadratic function of time. At times 1, 2, and 3 seconds after he fired it, the rock is 68, 96, and 114 meters above the ground, respectively.
   a. Write the particular equation for this function.
   b. What is the highest the rock will be above the ground?
   c. How high is the platform?
   d. The coefficient of the linear term in the equation is the initial upward velocity of the rock in meters per second. How fast was the rock going up at the instant Chuck fired it?
   e. On its way down, the rock goes into a well, splashing into the water exactly 10 seconds after Chuck fired it. How deep is the well?

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Test 13 | CHAPTERS 1 through 5

(2 hours)

Algebra/Trig is a study of functions and relations. Write the general equation for

1. a linear function.

2. a quadratic function.

Certain axioms form the basis for algebra. Tell which axiom is illustrated.

3. $a(b + c) = ab + ac$

4. $a + (b + c) = (a + b) + c$

5. $1 \cdot (b + c) = (b + c)$

6. If $a = b + c$, then $b + c = a$.

7. Show by example why {integers} is *not* closed under division.

Axioms, properties, and definitions can be used to solve equations and inequalities.

8. Solve and graph: $|3x - 7| \leq 11$, domain = {integers}

9. Solve: $(3x - 7)(x + 4) = 0$

10. Solve: $4x^2 - 12x + 13 = 0$, domain = {complex numbers}

You must know certain names and definitions.

11. Write the definition of a relation.

12. Write a cubic binomial.

13. Write an expression that is *not* a polynomial.

14. Write the discriminant of $rx^2 + sx + t = 0$.

15. Write a negative rational number that is not an integer.

The axioms and properties can be used to transform expressions.

16. Simplify: $13 - 2 \cdot 12 \div 6 + 5$

17. Simplify: $7 - 3[p - 2(5p - 6)]$

18. Multiply: $(3x - 4)(2x + 5)$

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(Test 13 continued)

19. Factor: $2x^2 - 11x + 12$

Graphs of familiar functions can be drawn by finding special points.

20. For $y = x^2 + 2x - 8$, find the vertex, find the y-intercept, find the x-intercepts, and sketch the graph.

21. Sketch the graph: $y - 3 = -\frac{5}{2}(x + 4)$

22. Sketch in 3 dimensions: $21x + 70y + 30z = 210$

Unfamiliar graphs can be drawn by pointwise plotting.

23. Plot $y = \sqrt{x}$, Domain: $0 \le x < 9$

24. Sketch the graph of a relation that is clearly *not* a function.

Functions can be used as mathematical models.

25. Allen Wilkins shoots a free throw in basketball. Sketch a reasonable graph showing its distance above the gym floor as a function of time since he shot the ball.

Making a model involves finding the particular equation for a function.

26. Find the particular equation of the linear function whose graph goes through the point $(-3, 5)$, and is parallel to the graph of $y = 7x - 13$. Write the answer in slope-intercept form.

27. Find the particular equation of the quadratic function containing the three points $(1, 16)$, $(2, 15)$, and $(4, 1)$.

Using the model involves finding x or y when the other is given.

28. If $f(x) = 3x + 7$, find $f(-4)$.

29. If $g(x) = 5x^2 - 3x + 6$, find $g(7)$.

30. If $y = 3x^2 - 5x - 2$, find x when $y = 11$.

Systems of linear inequalities and equations can be solved and graphed.

31. Plot the graph of the system:

$$y < -3x + 7$$
$$2x - 3y \le 12$$
$$y \ge -8$$

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(Test 13 continued)

32. Write N_z, the *numerator* determinant for z, and evaluate it.

$$\begin{aligned} 3x + 5y + 2z &= -8 \\ 4x - y + 3z &= 7 \\ x + 2y + 5z &= 1 \end{aligned}$$

33. Solve by augmented matrices.

$$\begin{aligned} 4x + 7y &= 1 \\ 3x - 5y &= 11 \end{aligned}$$

Now it is time for you to show that you can put everything together to work a mathematical model problem.

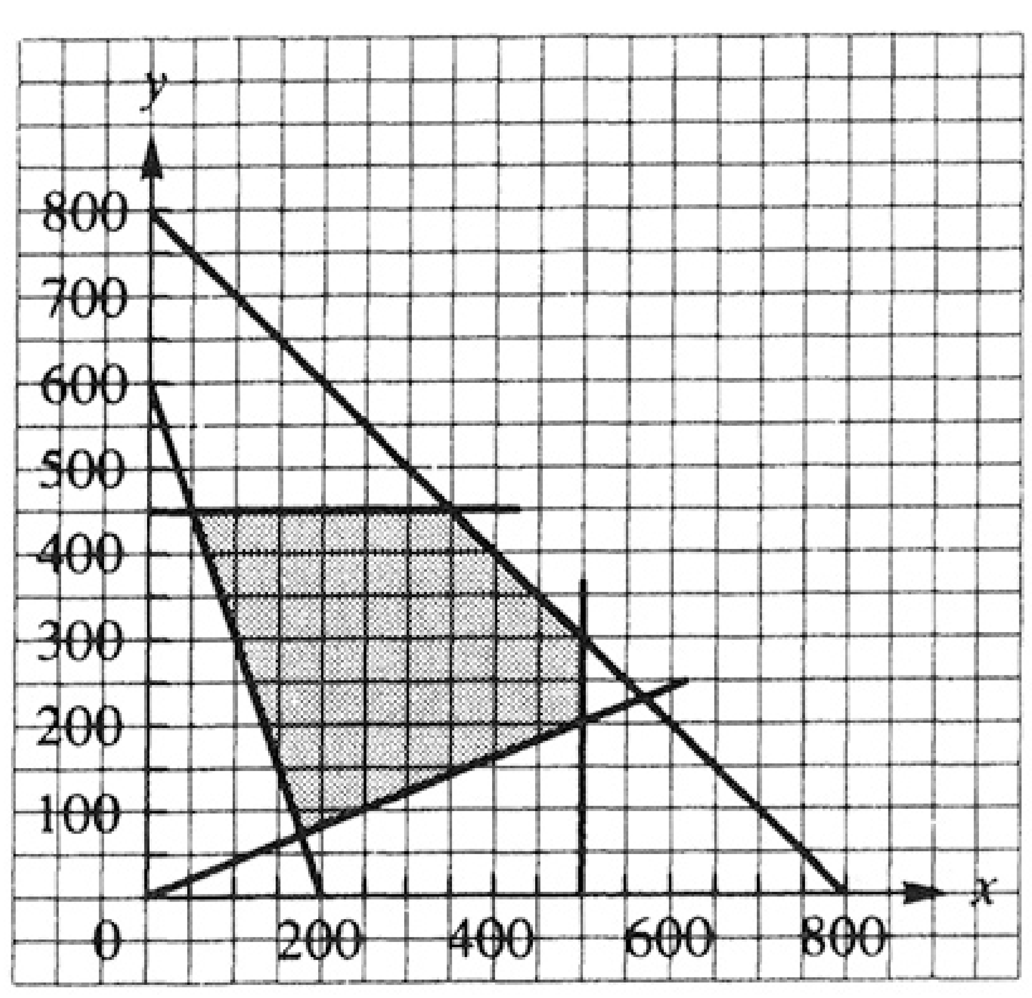
34. The number of dollars Wolf Photo charges you to develop a roll of film and make prints varies linearly with the number of good pictures that are printed. For a 12-print roll they charge \$7.33, and for a 15-print roll they charge \$8.29.

a. Write the particular equation expressing dollars in terms of number of prints.
b. How much would a 36-print roll cost?
c. If your bill is \$12.77, how many prints did they make?
d. What does the dollars-intercept represent in the real world?
e. What do they they charge *per print*? How do yóu tell?
f. Sketch the graph.

Systems of linear inequalities can be used to find the optimum point in a linear programming problem.

35. The graph shows the feasible region for the manufacture of two kinds of tennis shoe, Plain (x pairs), and Deluxe (y pairs). The profit on each pair of Plains is \$20 and the profit on each pair of Deluxes is \$30.

a. Write an equation for the total profit in terms of x and y.
b. Shade the portion of the feasible region in which the profits is at least \$15,000.
c. How many pairs of each should be produced to make the maximum profit?
d. What is the maximum feasible profit?



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Test 14 | CHAPTER 6, Sections 6-1 to 6-5

Algebra/Trig is a study of functions. Recently you have been studying exponential functions.

1. Write the general equation that defines an exponential function.

2. Sketch the graph of the exponential function $f(x) = 2^x$ for values of x from -3 to 3.

3. Given the exponential function $f(x) = 3 \cdot 5^x$, evaluate $f(3)$, $f(-2)$, and $f(0)$.

To deal effectively with exponential functions you must know the properties of exponents. Complete the following.

4. $(ab)^k =$ 5. $\dfrac{x^a}{x^b} =$ 6. $(y^u)^p =$

7. The words "exponent," "base," and "power" apply to the symbol x^y. Tell which word goes with which part of the symbol.

Calculators can be used to raise numbers to non-integer powers. Evaluate the following.

8. $2^{3.7}$ 9. $0.8^{-\frac{3}{7}}$

10. Explain why your answer to Problem 8 is *reasonable*.

Three definitions of exponentiation allow exponents to be zero, negative, or fractions. With these, you can operate on some powers *without* a calculator.

11. Use the definitions of exponentiation to show why $8^{-\frac{2}{3}} = 0.25$.

12. If $5^x = 625$, what does x equal?

13. Quick! What does 100^0 equal?

The properties of exponentiation let you transform expressions. Simplify the following.

14. $(4x^6y^{10})^3$ 15. $\sqrt[6]{(11^{7.5})(11^{-5.1})}$

16. $(6x^{\frac{2}{3}})(5x^{-\frac{1}{5}}y^7)$ 17. $\left(\dfrac{x^5}{y^6}\right)^7\left(\dfrac{y^8}{x^9}\right)^{10}$ 18. $\dfrac{3 \cdot 2001^{501}}{5 \cdot 2001^{500}}$

It is important to be able to work old problems.

19. Write the general equation of a quadratic function.

20. Draw a number-line graph: $14 - 3x > 29$

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Test 15 | CHAPTER 6, Sections 6-6 to 6-12

Algebra/Trig is a study of functions. Properties of exponential functions allow you to handle large and small numbers with scientific notation. Evaluate the following.

1. $(8.7 \times 10^{38})(9.324 \times 10^{-13})$ Round appropriately.

2. $\dfrac{3 \times 10^{871}}{5 \times 10^{265}}$ 3. $(3 \times 10^{-7})^4$ 4. $\dfrac{1}{2.5 \times 10^9}$

5. For the number 2.3×10^7, what special name is given to:
   a. the numeral 2.3?
   b. the numeral 7?

Logarithms allow you to find unknown exponents.

6. Solve for x: $4.86^{5x} = 3267$. Demonstrate that your answer is correct.

Logarithms have various properties.

7. Complete the following: $\log r^p =$

8. Demonstrate by calculator that $\log\left(\dfrac{57}{19}\right) = \log 57 - \log 19$.

9. Prove that $\log_b (rk) = \log_b r + \log_b k$.

10. Write as a single log of a single argument:
    $\log_3 24 + \log_3 17 - \log_3 51$

11. Suppose that $\log_b 5 = 1.234$. Quick! What does $\log_b 25$ equal?

Demonstrate that you know the definition of logarithm by using it to find the *exact* value of x. No decimal approximations!

12. $\log_3 x = 5$ 13. $\log_{32} 8 = x$ 14. $\log_x 64 = \dfrac{2}{3}$

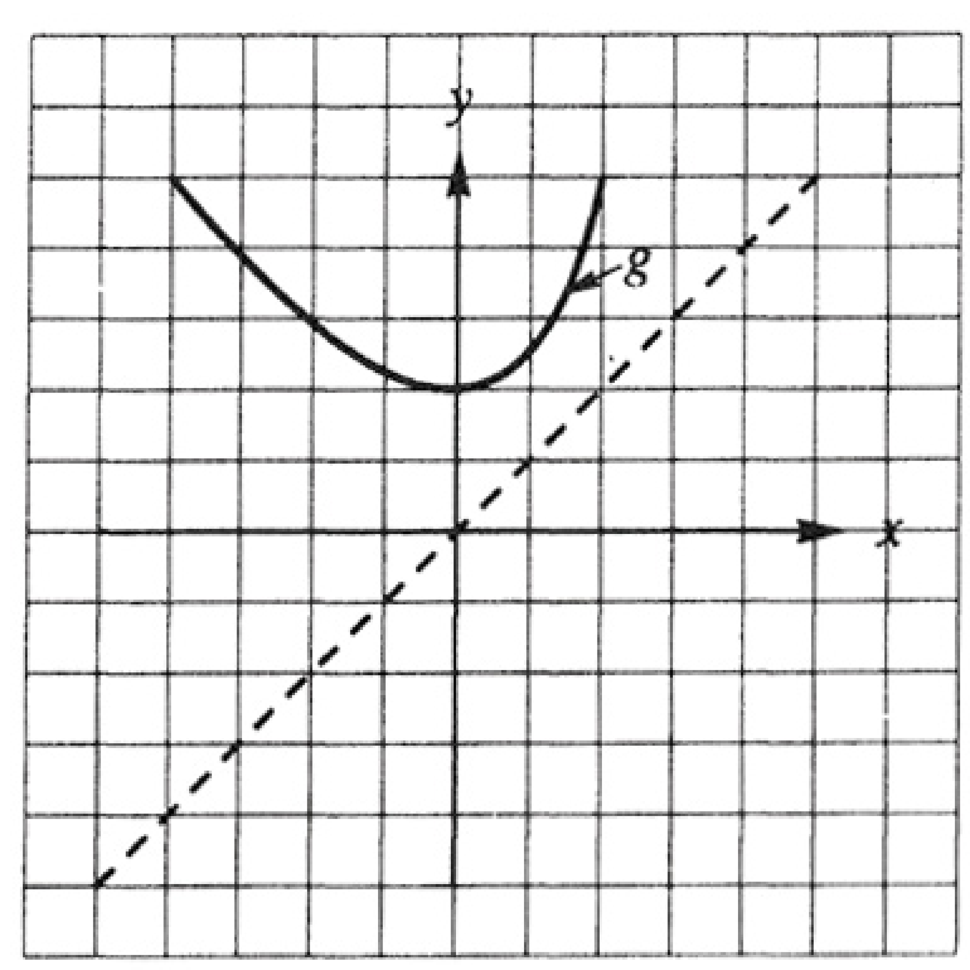
The definition of logarithm can be used to find logs to other bases.

15. Find $\log_7 451.9$.

Other functions besides exponentials have inverses.

16. Given the function $f(x) = 7x - 28$
    a. Write an equation for $f^{-1}(x)$ in terms of x.
    b. Evaluate $f^{-1}(35)$.
    c. Evaluate $f(11)$.
    d. Show that $f(f^{-1}(x)) = x$.

17. The sketch shows the graph of function g.
    a. Draw the graph of g^{-1}.
    b. Tell why g^{-1} is *not* a function.



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Test 16 | CHAPTER 6, Sections 6-13 and 6-14

Algebra/Trig is a study of functions. Recently you have used exponential functions as mathematical models. Doing so involves logarithms. Demonstrate that you know the three properties of logarithms by transforming the following. Do *not* evaluate!

1. $\log(3 \cdot 4) =$ **2.** $\log 3^4 =$ **3.** $\log\left(\frac{3}{4}\right) =$

Logarithms can be used to solve exponential equations.

4. Solve $3.7^{4x} = 9271$

The add-multiply property of exponential functions can sometimes be used to find values *without* finding the particular equation.

5. If f is an exponential function, and $f(5) = 20$ and $f(9) = 14$, use the add-multiply property to calculate $f(13)$, $f(17)$, and $f(21)$. Show your method.

6. Draw the graph of the function in Problem 5.

7. Show that the following data could *not* possibly be fit by an exponential function:

x	y
3	72
5	200
7	392

8. Show that y in Problem 7 varies directly with the square of x.

9. If $g(x)$ varies exponentially with x, and $g(0) = 37$ and $g(7) = 5940$, find the particular equation.

10. If $h(t) = 17.4 \times 1.23^t$, find $h(13)$.

11. If $z(t) = 60 \times 1.03^t$, find t when $z(t) = 144$.

An entire mathematical model application involves getting information from the words of the problem.

12. *Water Hyacinth Problem* The number of water hyacinth plants growing on a lake increases exponentially with time. On Tuesday the number of plants is 150. That Friday the number has risen to 240. If it takes 3400 plants to completely cover the lake, will be the lake be completely covered by 20 days after Tuesday? Justify your answer.

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Test 17 | CHAPTER 6

Algebra/Trig is a study of functions. This chapter concerns exponential functions and their inverses, logarithmic functions.

1. Given $f(x) = x^3$ and $g(x) = 3^x$. One of the functions is an exponential function and the other is not. Which is which? How do you tell?

2. Evaluate: -7^4

3. Evaluate: $5 \cdot 8 + 2 \cdot 4^3$

4. Use variables to state the following:
   a. Definition of logarithm
   b. Property of a quotient of powers with equal bases.
   c. Property of the log of a product
   d. Definition of $x^{\frac{1}{n}}$.

5. Simplify: $(3r^5m^7)^4(64r^0m^{12})^{\frac{2}{3}}$

6. Evaluate $\sqrt[9]{32586.4}$

7. Evaluate in scientific notation. Round appropriately.

$$\frac{3.7495 \times 10^{-21}}{7.24 \times 10^{17}}$$

8. If $f(x) = 7^x$, write an equation expressing $f^{-1}(x)$ in terms of x.

9. Solve for x and simplify: $\log_{25}x = \frac{3}{2}$

10. Find $\log_6 37.9$.

11. Solve for x: $\log_x 64 = \frac{2}{3}$

12. Express as a single log of a single argument:

$$3 \log_5 12 - \log_5 36$$

13. Write the characteristic and mantissa of 8.75×10^{29}.

14. Suppose that $h(x)$ is an exponential function for which $h(2) = 24$ and $h(5) = 36$. Find $h(11)$.

15. Sketch the graph of $y = a \cdot b^x$ if $0 < b < 1$.

16. *Aspirin Problem* Assume that the concentration of aspirin in the blood system decreases exponentially with time after you take the tablets. At time $t = 0$ the concentration is 100 units, and at time $t = 40$ minutes the concentration is 73 units. Find the particular equation and use it to perdict the concentration at time $t = 95$ minutes.

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Test 18 | CHAPTER 7, Sections 7-1 to 7-5

Algebra/Trig is a study of functions. In order to deal effectively with rational algebraic functions you must be good at factoring polynomials. Factor the following completely.

1. $12a^3b^5 + 18ac$

2. $9x^2 - 36$

3. $x^2 + 14xy - 15y^2$

4. $4x^2 + 2x - 30$

5. $4z^2 + 25z + 6$

6. $9x^2 - 48x + 64$

7. $x^3 + c^3$

8. $8p^3 - f^3$

9. $(x + a)y + (x + a)(3p - 4)$

Sometimes it is important to know whether or not a polynomial factors without actually finding the factors.

10. Prove that $36x^2 + 72x + 25$ is prime.

Factoring is the reverse process of multiplication. Multiply the following.

11. $(3x - 7)(4x + 8)$

12. $(x^2 - 7x + 3)(x - 4)$

13. $(5x + 11)^2$

14. $(x - 3)(x + 5)(x - 8)$

Long divide. Write the answer in mixed-number (or polynomial) form.

15. $\dfrac{4x^3 - 3x^2 + 7x - 19}{x - 2}$

16. $\dfrac{x^3 - 11}{x + 3}$

Sketch the graph, showing especially the behavior where the denominator equals zero.

17. $f(x) = \dfrac{1}{x - 3}$

18. $g(x) = \dfrac{(x + 2)(x - 3)}{x + 2}$

It is important to remember old concepts.

19. Sketch the graph of a quadratic function with a positive x^2-coefficient and a negative y-intercept.

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Test 19 | CHAPTER 7, Sections 7-6 to 7-9

Algebra/Trig is a study of functions. In order to deal effectively with rational algebraic functions you must be good at operating with algebraic fractions. Perform the indicated operations and simplify.

1. $\dfrac{x^2 - 64}{x^2 - 16} \div \dfrac{x - 8}{x - 4}$

2. $\dfrac{x - 7}{7} - \dfrac{x - 4}{4}$

3. $\dfrac{x}{x + y} + \dfrac{y}{x - y}$

4. $\dfrac{5x + 17}{x^2 + 8x + 7} - \dfrac{3}{x + 7}$

5. $\dfrac{20 + \dfrac{10}{x}}{20 - \dfrac{5}{x^2}}$

To work effectively with rational expressions you must be able to factor polynomials.

6. Use the Factor Theorem to factor $x^3 - 5x^2 - 2x + 24$. Factor further, if possible.

7. Given $P(x) = 3x^3 + 13x^2 - 4x - 4$:
   a. show that $P\left(\dfrac{2}{3}\right) = 0$.
   b. Write a linear factor with integer coefficients. (It is not necessary to find the other factor.)

8. Factor: $x^7 - y^7$

9. Factor: $a^3 + b^3$

The purpose for operating with algebraic fractions is to be able to plot graphs of rational functions. Plot the following, showing in each case what happens at each place the denominator equals zero.

10. $f(x) = \dfrac{5x - 10}{x^2 + x - 6}$

11. $f(x) = \dfrac{x^2 - 3x - 4}{x + 1}$

It is important to keep up your skill with mathematical models.

12. Windy's sells large orders of french fries (5 3/4 ounces) for 89 cents and medium orders (4 ounces) for 75 cents. Assuming that the number of cents is a linear function of the number of ounces, write the particular equation. If a giant order cost $1.47, predict the number of ounces it would contain. Sketch the graph.

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Test 20 | CHAPTER 7, Sections 7-10 and 7-11

Algebra/Trig is a study of functions. You have now studied variation functions, some of which are rational functions.

1. Write the general equation of a direct fifth-power variation function.

2. Sketch the graph of an inverse square variation function.

3. If p varies inversely with r, and $p = 37$ when $r = 54$, write the particular equation.

4. A variation function has $f(2) = 5$ and $f(6) = 45$. Which kind of variation function is it? Write the particular equation.

Variation functions can be used as mathematical models.

5. *Height-Weight Problem* The weights of people of the same proportions vary directly with the *cube* of the height. Phoebe Small is 5′ (i.e., 60″) tall and weighs 99 pounds.

a. Write the particular equation expressing weight as a function of height.

b. Predict the weight of a girl who is 6′4″ (i.e., 76″) tall, but who has the same proportions as Phoebe.

c. If Phoebe's little sister has the same proportions, but weighs only 80 pounds, how tall is she?

In working with rational fuctions you must sometimes solve fractional equations. Solve, discarding any extraneous solutions.

6. $1 + \dfrac{x}{x-3} = \dfrac{5}{x-3}$

7. $\dfrac{5}{x+1} + \dfrac{x}{x-2} = \dfrac{8x-7}{x^2-x-2}$

The purpose for operating with algebraic fractions is to facilitate plotting of rational function graphs.

8. Plot the graph, showing all critical features.

$$f(x) = \frac{12(x-1)(x-2)}{(x-1)(x+1)(x-4)}$$

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Test 21 | CHAPTER 7

Algebra/Trig is a study of functions. Operating with rational functions involves simplifying algebraic fractions. Simplify these.

1. $\dfrac{(x + 3)(x - 5)}{(x - 4)(x - 1)} \div \dfrac{(x + 3)(5 - x)}{(x + 4)(x + 7)}$

2. $\dfrac{x}{x - 2} - \dfrac{3}{x + 4}$

3. $\dfrac{x^5 + y^5}{x + y}$

4. $\dfrac{x + \dfrac{2}{x + 3}}{2 + \dfrac{x}{x + 3}}$

5. Multiply: $(3x - 7)(x^2 - 4x + 5)$

Simplifying rational expressions involves factoring.

6. Factor completely: $x^3 + 125$

7. Factor completely: $64x^2 - 16$

8. Given: $P(x) = x^3 - 5x^2 - 8x + 48$
   a. Show that $P(-3) = 0$.
   b. Factor $P(x)$ completely.

An "improper" algebraic fraction can be transformed by long division.

9. Write in mixed-number form:

$$\frac{3x^3 - 14x^2 - 29x + 5}{x - 6}$$

Operation with rational functions sometimes involves solving fractional equations.

10. Solve: $x + \dfrac{1}{x - 1} = \dfrac{x}{x - 1}$

Variation functions can be used as mathematical models.

11. Write the general equation for
    a. y varies inversely with the square of x.
    b. y is directly proportional to the cube of x.

12. Find the particular equation if y varies directly with x, and $y = 63$ when $x = 9$.

It is important to remember other kinds of functions.

13. Find the particular equation of the exponential function containing (0, 234) and (6, 5810).

A purpose for operating with algebraic fractions is to facilitate plotting of rational function graphs.

14. Plot the graph, showing all critical features.

$$f(x) = \frac{12(x - 1)}{(x - 1)(x + 2)}$$

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Test 22 | CHAPTER 8

Algebra/Trig is a study of functions. Irrational algebraic functions involve radicals or non-integer exponents.

Write in simple *radical* form:

1. $\sqrt{18} + 2\sqrt{50} - \sqrt{98}$

2. Simplify: $\dfrac{21}{\sqrt{12} + \sqrt{5}}$

3. Simplify: $\dfrac{1 - \dfrac{1}{\sqrt{3}}}{1 + \dfrac{1}{\sqrt{3}}}$

Solving radical equations sometimes leads to extraneous solutions. Solve and check.

4. $\sqrt{x + 3} + \sqrt{x - 2} = 5$

Variation functions can involve non-integer exponents.

5. *Labor Problem* Assume that the length of time it takes to do a major construction project varies inversely with the 0.7 power of the number of workers on the project. Suppose that 100 workers can construct an office building in 250 days. Write the particular equation expressing time in terms of number of workers, and use it to predict the number of workers needed to construct the building in just 180 days.

Variation functions can involve more than two variables.

6. *Bridge Problem* Building a 200-foot-long bridge across Scorpion Gulch, where the water is 8 feet deep, costs \$700,000. Assume that the cost varies directly with the length of the bridge and directly with the square of the depth of the water. Write the particular equation expressing cost in terms of length and depth. Use the equation to predict the cost of a bridge downstream from the present one, where the water is only 5 feet deep, but the length must be 600 feet.

Graphs of irrational functions show things about extraneous solutions. Let $f(x) = x - \sqrt{x - 2}$.

7. Find $f(11)$, $f(6)$, $f(3)$, $f(2.5)$, $f(2)$, and $f(0)$.

8. Plot the graph of f.

9. What is the domain of x if $f(x)$ is to be a *real* number?

10. Show that there are *no* real values of x for which $f(x) = 0$.

11. Set $f(x) = 4$, and solve for x. Show on your graph that one solution is valid, but the other one is extraneous.

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Test 23 | CHAPTERS 6 through 8

(2 hours)

Algebra/Trig is a study of functions. This test measures your knowledge of exponential, rational, and irrational functions, and refreshes your memory about linears and quadratics.

1. Sketch the graph of
- a. a decreasing exponential function.
- b. an inverse variation function.
- c. a rational algebraic function with vertical asymptote at $x = -2$ and removable discontinuity at $x = 4$.
- d. an increasing linear function.
- e. a quadratic function with negative x^2-coefficient.

2. Write the general equation for a function that has the indicated property.
- a. Quadrupling x makes y 64 times as big.
- b. Halving x makes y 4 times as big.
- c. Adding a constant to x multiplies y by a constant.
- d. Adding a constant to x adds a constant to y.

3. Multiply and simplify.
- a. $(\sqrt{8} + \sqrt{3})(\sqrt{15} + \sqrt{2})$
- b. $(\sqrt{3} - \sqrt{5})^2$
- c. $(\sqrt{y} - 6)(\sqrt{y} + 6)$
- d. $(3x + 11)(2x - 7)$
- e. $(x + 3)(x^2 - 4x - 5)$

4. Factor completely.
- a. $a^5 + b^5$
- b. $5x^2 + 27x + 36$
- c. $x^3 + 2x^2 - 11x - 12$
- d. $64p^2 - 36$

5. Carry out the indicated operations and simplify.
- a. $\dfrac{1}{x^2 + 4x + 3} + \dfrac{1}{x^2 - 1}$
- b. $\dfrac{9 - x^2}{3x + 9} \div \dfrac{x^2 - 5x + 6}{x^2 + 2x + 1}$

6. Simplify the following. *No* decimal approximations!
- a. $5x^{-\frac{2}{5}} \cdot 7x^{\frac{3}{4}}$
- b. $\sqrt{125} \div \sqrt[3]{25}$
- c. $\dfrac{12}{\sqrt{19} + 4}$
- d. $\dfrac{24}{\sqrt[3]{2}}$

(Test 23 continued)

e. $\sqrt[6]{36}$

f. $7\sqrt{45} + \dfrac{10}{\sqrt{5}}$

g. $\log_9 27$

h. $\log_7 30 + 2\log_7 5$

i. $\left(\dfrac{5}{x-2} + 1\right) \div \left(\dfrac{5}{x+3} - 1\right)$

7. Solve the equation.

a. $3^{2x} = 457$

b. $2 + \sqrt{x - 2} = x$

c. $x + \dfrac{1}{x-2} = \dfrac{x-3}{2-x}$

8. Given $f(x) = \dfrac{x^3 + 2x^2 - 11x - 12}{x^2 - x - 6}$,
tell the ordered pair at which there is a removable discontinuity.

9. *Bacteria Problem* The number of bacteria in a bottle of milk increases exponentially with time. Assume that there are 56 million bacteria at time $t = 0$ days, and 204 million at $t = 2$ days. Find the particular equation and use it to predict the number of bacteria at $t = 7$.

10. *Braking Distance Problem* The number of feet it takes your car to stop after you apply the brakes varies directly with the square of your speed and inversely with the coefficient of friction between your tires and the roadway. Assume that on a dry road the coefficient of friction is 0.8, and it takes 45 feet to stop when the speed is 30 miles per hour.

a. Write the particular equation expressing braking distance in terms of speed and coefficient of friction.

b. Lisa Carr left 290 feet of skid marks in coming to a stop on a wet road where the coefficient of friction was only 0.3. Predict her speed when she applied the brakes.

11. *Christmas Tree Problem* Data on the price of Christmas trees from Maple Grove Fruit Farm in Montague, Michigan, show that the price is directly proportional to the 2.3 power of the height of the tree. A 6-foot Douglas fir costs $48.00. Write the particular equation expressing price in terms of height. Use the equation to predict the price of a 60-foot tree such as might be used at the White House.

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Test 24 | CHAPTER 9, Sections 9-1 to 9-5

Algebra/Trig is a study of functions and relations. Quadratic relations have graphs that are conic sections.

Transform the equation or inequality by completing the square. Sketch the graph.

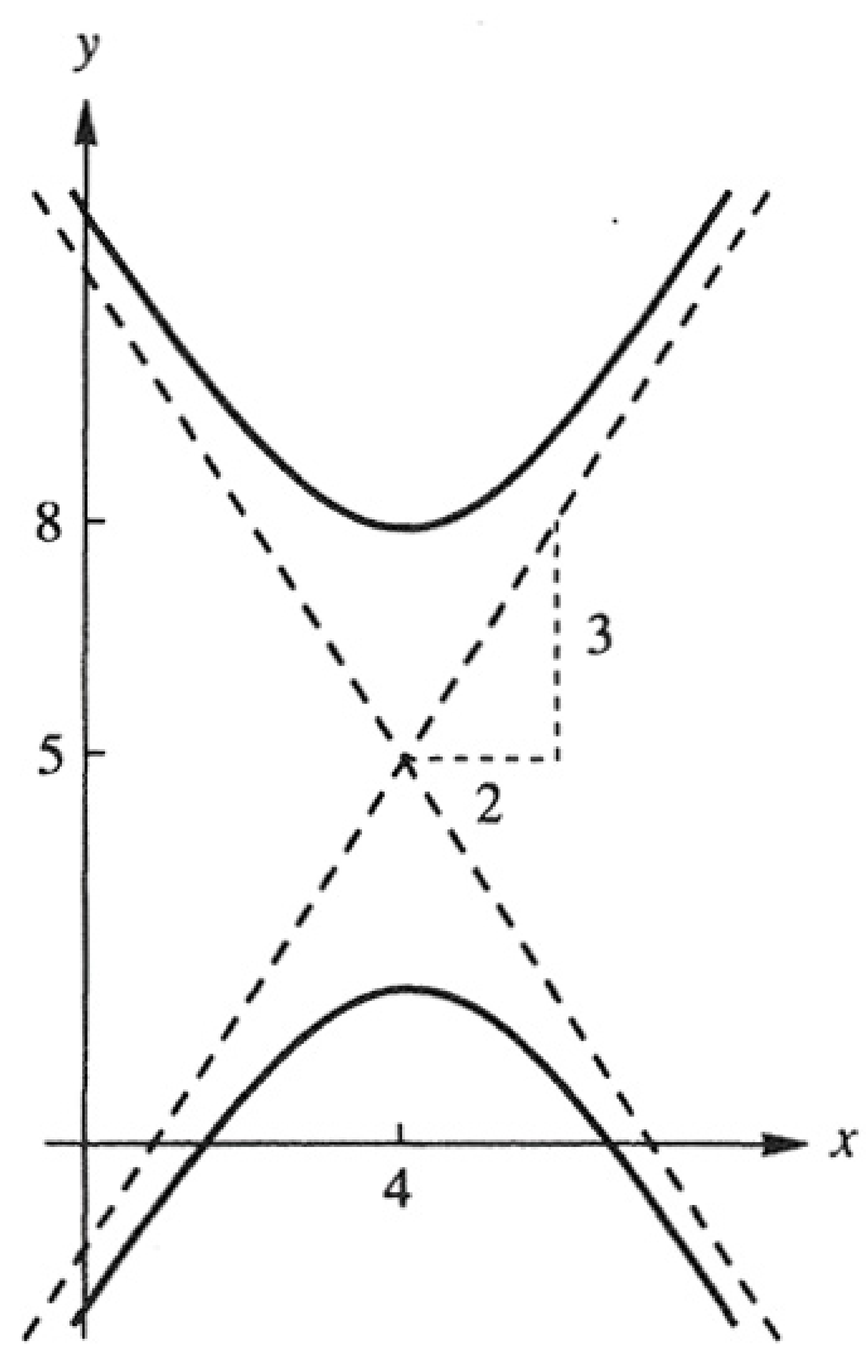
1. $x^2 + y^2 + 4x - 10y + 20 < 0$

2. $4x^2 + y^2 - 8x + 6y - 3 = 0$
(Also, show the foci.)

3. $y^2 + 2x - 6y - 7 = 0$

You can reverse the graphing process and find the equation for a given conic.

4. Write an equation for the hyperbola sketched. Transform to the form $Ax^2 + Cy^2 + Dx + Ey + F = 0$, where A, C, D, E, and F are integer constants.



It is important to remember old techniques.

5. Solve: $\dfrac{-9}{x-2} - x = 8$

6. Solve: $\sqrt{x^2 + 35} - x = -7$

7. Simplify: $\dfrac{22}{\sqrt{18} + \sqrt{7}}$

Addison-Wesley Publishing Company

ISBN 0-201-25288-0
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